EFFECTIVE TEACHERS OF NUMERACY

Mike Askew, Valerie Rhodes, Margaret Brown, Dylan Wiliam and David Johnson

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Effective Teachers of Numeracy: Summary

Summary of Findings

This project explores the knowledge, beliefs and practices of a sample of effective teachers of numeracy. It is one of a small number of projects where effectiveness is defined on the basis of learning gains: i.e. teachers were identified as highly effective if their classes of pupils had, during the year, achieved a high average gain in numeracy in comparison with other classes from the same year group.

i. What distinguished highly effective teachers from other teachers was a particular set of coherent beliefs and understandings, which underpinned their teaching of numeracy. Their beliefs focused upon:

- what it means to be numerate
- the relationship between teaching and pupils' learning of numeracy
- presentation and intervention strategies.

The beliefs determined, for example, what type of questions teachers asked and how they followed them up, irrespective of whether they were talking to pupils individually, in a group or in the whole class.

ii. Highly effective teachers believed that being numerate requires:

- having a rich network of connections between different mathematical ideas
- being able to select and use strategies, which are both efficient and effective.

They used corresponding teaching approaches that:

- connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols and diagrams
- used pupils' descriptions of their methods and their reasoning to help establish and emphasise connections and address misconceptions
- emphasised the importance of using mental, written, part-written or electronic methods of calculation that are the most efficient for the problem in hand
- particularly emphasised the development of mental skills.

iii. Highly effective teachers believed in relation to pupils' learning that:

- almost all pupils are able to become numerate
- pupils develop strategies and networks of ideas by being challenged to think, through explaining, listening and problem solving.

They used teaching approaches that:

- ensured that all pupils were being challenged and stretched, not just those who were more able
- built upon pupils' own mental strategies for calculating, and helped them to become more efficient.

iv. Highly effective teachers believed, in relation to teaching that:

- discussion of concepts and images is important in exemplifying the teacher's network of knowledge and skills and in revealing pupils' thinking
- it is the teacher's responsibility to intervene to assist the pupil to become more efficient in the use of calculating strategies.

These teachers used teaching approaches that encouraged discussion, in whole classes, small groups, or with individual pupils.

4. One group of less effective teachers of numeracy believed in the importance of pupils acquiring a collection of facts and standard methods, and that pupils varied in their ability to remember these. They used teaching approaches that:
   - dealt with areas of mathematics discretely
   - emphasised teaching and practising standard methods and applying these to abstract or word problems without considering whether there were alternative more efficient ways of solving a particular problem.

5. A second group of less effective teachers believed in the importance of developing numeracy concepts using practical equipment and waiting until pupils were ready to move onto more formal methods. They used teaching approaches that emphasised pupils working things out for themselves, using any method with which they felt comfortable.

6. Some teachers combined some characteristics of highly effective and less effective teachers. The classes taught by such teachers had gains that were between those of the highly effective and less effective groups.

7. Highly effective teachers of numeracy had knowledge and awareness of inter-relations between the areas that they taught of the primary mathematics curriculum. Being highly effective was not associated with having an A-level or degree in mathematics. Less effective teachers of numeracy, including some with mathematics qualifications, displayed knowledge that was more compartmentalised and in some cases were reliant on procedures.

8. Highly effective teachers of numeracy used systematic assessment and recording methods to monitor pupils' progress and record their strategies for calculation, to inform planning and teaching. Less effective teachers either used little assessment or used it as a check that taught methods had been learned.

9. The mathematical and pedagogical purposes behind particular classroom practices are as important as the practices themselves in determining effectiveness. For example both highly and less effective teachers demonstrated a range of classroom organisation styles, and sometimes used mental tests with the whole class.

10. Highly effective teachers were much more likely than other teachers to have undertaken mathematics-specific continuing professional development over an extended period, and generally perceived this to be a significant factor in their development.

11. In some schools, experienced and highly effective staff were able, over time, to make other teachers more effective through working closely with them in detailed planning and evaluation, and working together in the classroom.

Aims and overall rationale of the study

The aims of the study *Effective Teachers of Numeracy* were to identify:

1. key factors which enable teachers to put effective teaching of numeracy into practice in the primary phase
2. strategies that would enable those factors to be more widely applied.

The working definition of numeracy used by the project was a broad one:

> *Numeracy is the ability to process, communicate and interpret numerical information in a variety of contexts.*

The identification of effective teachers of numeracy was based on increases in pupil attainment on specially designed tests of numeracy, administered by teachers.

Having assessed pupils at the beginning of the Autumn term and again at the end of the Spring term the gains in pupils scores were compared for particular teachers and used to identify which teachers, on the basis of the tests, were the most highly effective.

For these teachers it was possible to explore how their classroom practice in teaching numeracy was influenced both by their beliefs and their knowledge. This was then compared with the beliefs and knowledge of other teachers who were not so highly effective.

The framework for analysis

We examined three aspects of beliefs that might influence the teaching of numeracy:

- *Beliefs about the nature of numeracy*. This includes teachers' beliefs about:
  - –the nature of numeracy
  - –expectations of learning outcomes.
- *Beliefs about pupils and how they learn to become numerate***.** Included here are beliefs about:
  - –whether or not some pupils are naturally more mathematical
  - –the type of experiences that best bring about learning
  - –the role of the pupils in lessons.
- *Beliefs about how best to teach pupils to become numerate*. These are related to beliefs about teaching numeracy in terms of:
  - –perception of the teachers' role in lessons;
  - –the influence of the 'accepted' wisdom of 'good' primary practice.

Three areas of knowledge were also explored:

- *Numeracy subject knowledge*: understanding of mathematics and numeracy content knowledge appropriate to what is being taught.
- *Knowledge of pupils*: what particular pupils currently being taught understand as well as knowledge of pupils more generally, for example aspects of the numeracy curriculum that are generally found difficult.
- *Knowledge of numeracy approaches and representations:* understanding of ways of working with pupils and ways of presenting numeracy to pupils so that they gain access to the subject knowledge.

Methods

Evidence was gathered from a sample of 90 teachers and over 2000 pupils on what the teachers believed, knew, understood and did and outcomes in terms of pupil learning.

From an initial sample size of all the primary schools in three local education authorities (some 587 schools), together with Independent (private) schools, eleven schools were selected, providing a sample of 90 teachers. We selected the majority of these eleven schools on the basis of available evidence suggesting that the teaching of mathematics in these schools was already effective.

A specially designed test ('tiered' for different age ranges) of numeracy was administered to the classes of these 90 teachers, first towards the beginning of the autumn term 1995, and again at the end of the spring term 1996. Average gains were calculated for each class, providing an indicator of 'teacher effectiveness' for the teachers in our sample.

In order to broadly classify the relative gains, the teachers were grouped into three categories of highly effective, effective, or moderately effective. This classification was made by putting the classes in rank order within year groups according to the average gains made.

From the sample of 90 teachers the research team worked more closely with 33 teachers. Intensive work with 18 of these teachers (the case study teachers) provided data over two terms (Autumn 1995 and Spring 1996) on classroom practices together with data on teacher beliefs about, and knowledge of, mathematics, pupils and teaching.

The further 15 teachers (the validation teachers) were also observed and interviewed in the summer term 1996, although less intensively than the case study teachers. This data was used to supplement and check out hypotheses made on the basis of the case-study data and the whole sample.

Five methods were selected and developed to provide data on teachers':

- beliefs
- knowledge
- professional development experiences
- practices

Questionnaire: A questionnaire was administered to all 90 teachers.

Classroom observations: In total, 84 lessons were observed, three for each of the case study teachers and two for each of the validation teachers.

Head teacher interviews: Interviews were carried out with the 6 head teachers of the schools from which the case study teachers were selected.

Teacher interviews: 54 interviews with case study teachers were carried out, three for each of the 18 teachers. Specific interview instruments were developed to assess numeracy content knowledge and knowledge about pupils numeracy.

Validation teacher interviews: Each of the fifteen validation teachers was also interviewed on two occasions.

Chapter 1: Identifying Effective Teachers of Numeracy

1.1 Introduction

There is currently much concern about national standards of numeracy. This arises partly from comparatively poor performance in numeracy in international surveys. There is also some evidence of decline in performance over time (Keys, Harris, & Fernandes, 1996; Reynolds & Farrell, 1996).

While concern has been expressed at all levels from early years to undergraduate intake, Ofsted focus on the later years in primary schools, reporting that:

> 'In Key Stage 2, mathematics is judged to be the weakest subject in the curriculum...Pupils' understanding of mathematics is judged to be particularly weak in half of all schools... Teachers have little theoretical understanding about how progress in learning number occurs...Immediate benefit would be seen if teachers' confidence in their own mathematical competence could be improved.'
>
> (Office for Standards in Education, 1994).

The Teacher Training Agency (TTA) shared this concern of Ofsted and as a result the TTA responded by financing a study relating to the professional development of teachers of numeracy in primary schools. This study, entitled *Effective Teachers of Numeracy*, was carried out by staff at the School of Education, King's College London in co-operation with teachers and advisers in three LEAs (Berkshire, Croydon and Wandsworth) and the Incorporated Association of Preparatory Schools.

1.2 What were the aims of the study?

The aims of the study *Effective Teachers of Numeracy* were to identify:

1. key factors which enable teachers to put effective teaching of numeracy into practice in the primary phase
2. strategies, which would enable those factors to be more widely applied.

This report provides evidence from a sample of 90 teachers and over 2000 pupils on what teachers know, understand and do, and the relation of this to the outcome in terms of pupil learning.

Before addressing these issues it is necessary to set out both how we defined *numeracy* and our interpretation of *effective teaching*.

1.3 How do we define numeracy?

The working definition of numeracy used by the project was a broad one:

> *Numeracy is the ability to process, communicate and interpret numerical information in a variety of contexts.*

Although this definition encompasses the ability accurately to carry out arithmetical calculations, it goes beyond that to include conceptual understanding of number, a 'feel for number', and the ability to apply arithmetic.

1.4 How do we define effective teachers?

Careful identification of teachers believed to be effective in teaching numeracy was crucial to this study. Building on our definition of numeracy we defined effective teachers of numeracy as teachers who help pupils:

- acquire knowledge of and facility with numbers, number relations and number operations based on an integrated network of understanding, techniques, strategies and application skills
- learn how to apply this knowledge of and facility with numbers, number relations and number operations in a variety of contexts.

Judgments about the effectiveness of teachers in bringing about the above learning outcomes may be based on expectations of and evidence about one or more of the following:

- teacher behaviour
- pupil behaviour
- pupil learning outcomes.

Identifying effective teachers of numeracy in terms of expectations of teacher behaviour.

The origins of notions of effective teacher behaviour are numerous. For example, teachers may respond to influences such as:

- what teacher trainers, local advisors and mathematics co-ordinators in school expect;
- what Ofsted recommends;
- what the media and literature suggest as 'good practice'.

For this research to be based on expectations of teacher behaviour it would have been necessary to identify what is known about what kind of teacher behaviour leads to sound pupil learning. To date, research on learning mathematics has largely been separate from research on teaching mathematics. Factual evidence about what actually works in terms of bringing about effective learning of numeracy is limited.

In the absence of evidence about specific behaviour which promotes learning, judgments about teachers' behaviour are likely to be unsubstantiated and determined according to the belief system of the observers (in this case members of the research team), based on what they perceive to be 'good' practice. This was a situation we and the TTA were keen to avoid.

Identifying effective teachers in terms of expectations of pupil behaviour.
Significant factors relating to effectiveness here would include taking into account not simply what the teacher does but how the pupils respond within lessons, how engaged pupils are with activities and evidence of how understanding appears to develop within the context of the lesson.

While this focus on pupils adds an important dimension to determining effective teaching it still presents difficulties. For example, learning cannot be observed directly, so 'proxies' such as 'time on task' are used as substitutes. Recent research demonstrates that time on task may be a poor proxy for measures of learning (Boaler, 1996). While pupils may demonstrate understanding of the mathematics within the context of the lesson, the question of the extent of continued understanding or understanding in other contexts remains open. Once again there is a need for evidence about what kinds of behaviour, this time pupils' behaviour, are indicators of sustained learning.

Identifying effective teachers in terms of pupil learning outcomes
The idea that effective teachers are those who bring about identified learning outcomes was our starting point for the project. *It was decided that as far as possible the identification of effective teachers of numeracy would be based not on presumptions of 'good practice' but on rigorous evidence of increases in pupil attainment*. We could then address the question of what practices appear to be most effective in promoting pupils' learning as judged by gains in levels of attainment on an appropriate assessment instrument.

Because our focus was at the level of teachers rather than schools, it was necessary to gather data on improvement in pupil and class performance between two test administrations within the same school year. We did this by administering specially designed tests of numeracy to whole classes of pupils, first towards the beginning of the autumn term 1995, and again at the end of the spring term 1996. The tests related as far as possible to the definition of numeracy and the outcomes of effective teaching given above. Average gains could then be calculated for each class, and so provide an indicator of 'teacher effectiveness' for the teachers in our sample.

This decision to rely on evidence of pupil performance was not without its difficulties. Waiting until after the second test administration would enable us to identify which classes had made the most gains but would leave it too late in a 1-year project to study in detail the knowledge, understanding and practices of the teachers of these classes. Where performance data already existed it related only to school performance, and not to the performance of individual classes and their teachers.

On the other hand, identifying teachers before the pupil gain results were known ran the risk that their pupils might not make especially good gains and that we might therefore have spent time working with a group of teachers who turned out to be less effective.

How then were we to maximise the likelihood that the sample of teachers with whom we worked would include some who were effective? Our solution was a two-stage approach to identifying a sample of effective teachers through:

- identifying a sample of *focus schools* which appeared to be effective in teaching mathematics
- identifying a sample of teachers, believed by head teachers and others to be effective, within these schools.

1.5 Identifying 'focus' schools

The *focus schools* were a group of schools identified on the basis of being already known to be performing well above expectations in relation to numeracy.

Three Local Education Authorities (Berkshire, Croydon and Wandsworth) were approached, each known already to hold considerable school-level data on standards in numeracy in relation to other school variables. On the basis of this data, each LEA agreed to assist in identifying two 'effective focus schools'.

In order to identify schools that were effective in terms of 'value-added' rather than just high scoring, we analysed LEA data that included IQ scores, reading test scores, baseline assessments and National Test results. On the basis of this analysis, no schools emerged as particularly outstanding in 'value-added' terms but we were nevertheless able to select schools from a group identified as effective. Priority was given to including focus schools with more than one teacher per age group in order to enable some differentiation between the effect of a school and the effect of an individual teacher. We also checked that the sample contained schools with different socio-economic intakes in different environments (inner city, suburban, rural).

In order to include teachers in independent schools, we also approached the Incorporated Association of Preparatory Schools, and were assisted in identifying two further focus schools in the independent sector within the Home Counties that were acknowledged to be effective in teaching numeracy.

Because of other pressures, two of the eight schools initially selected as focus schools, each from a different LEA, had to withdraw from the study. This left in the study six focus schools. (Details of the six focus schools are given in Appendix 1.1).

Teachers in these six focus schools would provide the main sample of teachers (66 in all). Data from these teachers, matched against the pupil gains achieved, would help to identify 'key factors' characterising effective teachers of numeracy.

A sub-sample of teachers from each school, selected as likely to be particularly effective, would provide additional detailed case-study material (see 1.7 below for further details about the selection of the 'case-study' teachers).

Having checked with the test results towards the end of the year which of these case study teachers were indeed the most effective in terms of pupil gains in numeracy, we would be able to use this case study data to provide a fuller account and deeper understanding of the key factors characterising effective teachers of numeracy.

However there would still be some way to go before concluding that these factors were the significant ones. How were we to know that any factors identified were common to other effective numeracy teachers in other schools? And that such factors were not shared by any less effective teachers? In other words, how were we to know our findings were valid?

Our check on validity was to test out the findings from the focus schools with a variety of other teachers in a varied group of other schools, from the same LEAs and in the independent sector. Hence a second set of schools was designated *validation schools*; this second sample of five schools was identified as likely to include between them a variety of effective, average and less effective teaching in mathematics.

1.6 Identifying 'validation' schools

Whereas the 'focus' schools were selected on the basis of good ('value-added') attainment in mathematics, the group of *validation schools* was chosen to demonstrate a range of performance in mathematics.

To provide this variety, and particularly to include some smaller schools in the sample, five schools were selected as validation schools. In order to make this selection, judgments about school performance had to be made on the basis of available data including National Test and other results. These judgments about 'good', 'average' and 'weak' results should be interpreted not on an absolute scale but in relation to the available data on the intake and environment of the school.

- 1-form entry school with good mathematics and English results
- independent school with good mathematics and English results
- small village school with average results
- school with good English but average mathematics results
- school with weak mathematics and English results

Further details of the validation schools are given in Appendix 1.2.

Our selection of both *focus* and *validation* schools was not random but chosen on the basis of two criteria:

- evidence of performance in the teaching of mathematics;
- providing a representative range of schools in terms of size, socio-economic intakes and environments (inner city, suburban, rural).

1.7 Identifying a sample of teachers for more detailed case study

In order to provide the necessary level of detail in our data to understand the factors contributing to the development of effective teachers, it should be emphasised that our sample went through a series of progressive filters.

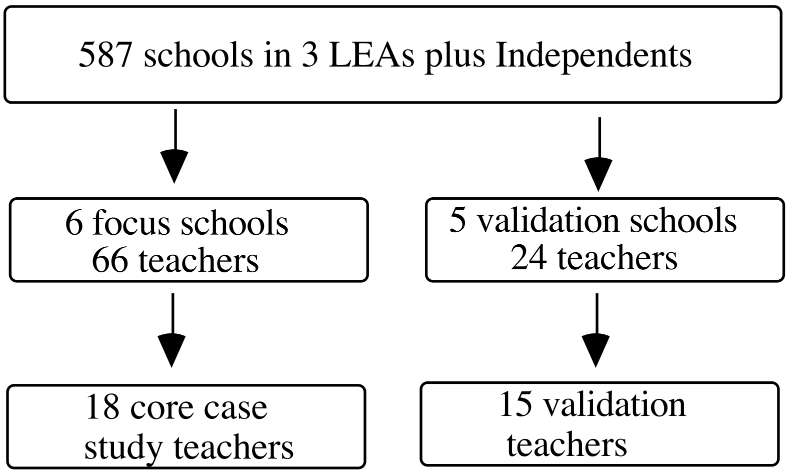

Figure 1.1 The sample of teachers

From an initial sample of all the primary schools in three LEAs (some 587 schools), together with Independent schools, we had selected eleven schools; six focus schools and five validation schools, to study in detail, giving an overall sample of 90 teachers.

Pupil test data (on our specially designed tests of numeracy, see section 1.8) and teacher questionnaire data was to be gathered for all 90 teachers in both core and validation schools. Further subsamples of 33 teachers (18 case study teachers from the 6 focus schools and 15 validation teachers from the 5 validation schools) within the 90 were selected to provide more detailed case study data.

From the six focus schools, we decided to work closely with 18 teachers, 3 in each school. This group of 18 teachers would form our sample of *case study teachers*, providing data on classroom practices, teacher beliefs about, and knowledge of, mathematics, pupils and teaching. The 3 teachers in each school were identified as those most likely to prove effective. Selection was done through discussion with head teachers in the focus schools and, where appropriate, with advice from the LEA inspectors and advisors. While the emphasis was on identifying effective teachers, the group of 18 was chosen so that teachers were reasonably evenly distributed across year groups 1-6.

Pupils in the validation schools were assessed at the beginning of the academic year and two terms later in exactly the same way and at the same time as the pupils in the focus schools. However, since the visits to the validation schools would not be until the summer term, details of class average gains would be available to assist in the selection of 15 *validation teachers* for closer study. Again, three teachers would be selected in each of the five schools for closer study. However, given the time scale of the work, less detailed data would be available on these teachers than on the case study teachers in the focus schools.

1.8 The Tests

In order to measure the gains in numeracy over the six month period October to April, a set of three 'tiered' tests for different age ranges (Years 1 and 2, Years 3and 4, Years 5 and 6) were developed and trialled. These were based on a diagnostic test assessing mental facility with numbers, and the ability to apply this, which had previously been designed and used at King's. The test had been shown to have high indices of validity and reliability, and the construct validity was further checked against the requirements of the national curriculum appropriate to each tier.

Aspects of numeracy which were covered in the tests were:

- Understanding of the number system, including place value, decimals and fractions
- Methods of computation, including both known number facts and efficient and accurate methods of calculating
- Solving numerical problems, including complex contextualised word problems and abstract mathematical problems concerning the relationships between operations.

An aural mode of testing was chosen where the teacher read out questions and pupils wrote down answers in specially designed answer books. This was done mainly to control the time pupils were allowed for each question; a wholly written test would not have enabled efficient methods to be so readily distinguished from more primitive time-consuming strategies based mainly on counting. Reading out questions was also more appropriate for younger children and weaker readers, enabled repetition where necessary, and maintained concentration for all groups. (It was decided that most of the year 1 pupils would only be included in the April testing round as some would not be sufficiently mature in the October.)

The tests were designed to be "fine grained" enough to demonstrate gains in attainment over two terms, while broad enough to be suitable to cover a wide range of levels of attainment. On the basis of the range of marks that pupils attained on each occasion that they took the tests, the assessments were judged as successful both in differentiating pupils' performance and in allowing progress to be measured.

Further details about the tests are given in Appendix 1.3.

The tests were marked and analysed at King's College; no data was fed back to schools between the two administrations of the tests.

Because it was harder for pupils to make high gains if their initial test score was close to the total, the scores were adjusted to take this 'ceiling effect' into account. (The adjustment is described in Appendix 1.3.)

Details of the final sample of classes tested are given in Table 1.1. Because some teachers in focus schools taught more than one class, sometimes in different year groups, there are fewer teachers than classes. In contrast, in some of the validation schools more than one teacher taught some classes. Equally, several classes, especially in the smaller validation schools, contained pupils of two or more Year groups.

	Number of classes	**Number of teachers**
Year 2 to Year 6 (tested twice)	86 (70 in focus schools, 16 in validation schools)	73 (54 in focus schools 19 in validation schools)
Year 1 (tested once)	17 (12 in focus schools, 5 in validation schools)	17 (12 in focus schools, 5 in validation schools)
Total	103	90

Table 1.1 Details of sample tested

Because of the greater difficulties in judging the success of Year 1 teachers, with in most cases only the final set of test scores, it was decided to refer for most of the analysis to teachers of Years 2 to 6 only.

1.9 The results of this approach to identifying and selecting effective teachers

This approach to the research enabled data from the teacher questionnaires to be directly related to a measure of effectiveness in terms of the gains in class mean test scores (adjusted). This was possible for the sample of all the 73 teachers of Year 2 to Year 6 in the focus and validation schools.

However for the purpose of identifying effective teachers of numeracy and contrasting their characteristics with those of other teachers by analysis of qualitative data, we decided that it would be simpler to group the 73 teachers into broad categories according to their relative effectiveness, as measured by mean adjusted class gains.

All the classes of the 73 teachers in Years 2-6 displayed increased mean class scores at the second time of testing. Few, if any, ineffective teachers had been anticipated in the sample, and in the event all classes showed reasonable gains. However, there was sufficient variation between the class gains for it to be clear that some of the teachers were more effective than others, as judged by class gains.

A broad classification of the relative gains for all of the 73 teachers of Years 2–6 (from both focus and validation schools) was made. In order to do this, the teachers were grouped into three categories of *highly effective*, *effective*, or *moderately effective* according to whether the mean gains of the classes they taught were high, medium or low relative to those of the other teachers in the study.

This classification was carried out by putting the classes in rank order within year groups according to the average (adjusted) gains made. This was done first within the groups of classes of pupils who took the same tests: Years 5 and 6 together, Years 3 and 4 together and Year 2 separate. The cut-off points between high, medium and low gains were decided on pragmatic grounds, so that classes in each year group fell into three roughly equal groups, but avoiding any situation where classes with nearly equal adjusted gains were allocated to different groups. The groups should not be interpreted as representing any predetermined quantitative differences between the classes based on expectations of what a 'medium' gain should be.

Table 1.2 presents the distribution of teachers in focus and validation schools in the three effectiveness categories, including the pseudonyms of 16 of the 18 case study teachers in focus schools. (Two case study teachers, Claire and Frances, are excluded from this table because they both taught Y1 for which gain scores were not available.) The initial of the pseudonym given to each teacher indicates which focus school they came from; for example Anne, Alan and Alice were the three case-study teachers in School A.

	Highly effective	Effective	Moderately effective
Focus schools:	Anne	Danielle	Brian
Case study teachers	Alan	Dorothy	Beth
	Alice	Eva	Cath
	Barbara	Fay	David
	Carole		Elizabeth
	Faith		Erica
Focus schools: non-case study teachers	17 teachers	11 teachers	11 teachers
Validation schools	3 teachers	6 teachers	10 teachers
TOTAL	*26 teachers*	*21 teachers*	*27 teachers*

Table 1.2: Teachers grouped according to relative mean class gains.

As can be seen, only six of the teachers we had selected for case study in the focus schools were placed in the highly effective category. This group of teachers had many common features, which were not shared by case study teachers in the moderately effective group. Hence the group, although smaller than expected, was thought large enough to act as a sample to enable key factors to be elicited. The findings were later supported by reference to the additional three highly effective teachers in the validation schools.

However it is interesting to note the large number of highly effective teachers who were not selected for case study. One reason for this was that nine of these other 17 teachers all came from School A, which already had three teachers in the highly effective group. In the case of other schools, some of the remaining eight highly effective teachers might have been selected by their heads but were not. Several possible reasons may account for this. The most likely reason is that some highly effective teachers were unwilling because of other pressures to take part in the case study work, which required additional interviews and observation.

Perhaps more surprisingly, several teachers, who had been selected as effective numeracy teachers in schools with good mathematics performance, turned out only to be moderately effective compared to the remaining teachers in their schools. Possible reasons for this include heads not recognising who were the most effective teachers of numeracy in the school, or perhaps either knowingly or unknowingly directing us to teachers for other reasons, such as firmness of class control.

The fact that our sample of case study teachers from focus schools spanned the highly effective, effective and moderately effective scale was helpful to the analysis in so far as the detailed data on these teachers helped to inform the contrast between more and less effective teachers.

As a result the data from validation schools was used mainly to supplement the more detailed qualitative data from the case-study teachers in focus schools. In all cases the data from the validation schools supported findings from the focus schools.

1.10 How does this sample compare with primary teachers in general?

Clearly the sample of schools participating in the study was biased towards schools effective in teaching numeracy. Eight of the eleven schools, all six focus schools and two of the five validation schools, were selected as effective on evidence of value-added performance (or, in the case of independent schools, expert opinion).

However in the case of state schools (8 out of the 11 schools) the data used for selection suggested that these were members of a large set of effective schools rather than members of an elite set of schools with outstanding results. The results of the study seemed to confirm that this was indeed generally the case.

The findings show that two schools (both state and both from the 6 focus schools) stood apart as performing better than the others. One of these schools achieved outstanding results, which put 12 of its 13 teachers in the highly effective category.

The mean gains for different classes in the remaining six schools selected as effective suggested that each contained at least one highly effective teacher and a small number of effective teachers. However both results and classroom observation suggested that about half the teachers in these remaining six effective schools seemed to be at best averagely competent in teaching numeracy, with a few probably below average.

This is confirmed by the data from the three other validation schools. Two of these were selected as being of only average effectiveness in their numeracy teaching using value-added criteria, with the third weak. Judged by the class gains in this study, none of these three schools was distinguishable from most of the eight schools selected as effective. Translating mean gains into teacher effectiveness, they contained roughly the same distribution of teachers between the three effectiveness categories as most of the group of effective schools. (The detailed data is in Table 5.1.)

In the case of the independent schools, the initial data available was very limited, so the selection of three effective schools was based mainly on expert advice. It would seem that in comparison with most of the state schools in the sample, two were slightly more effective, while the third was less effective than any of the state schools. None of the three was as effective as were the best two state schools.

The finding that two schools suggested by LEA-held value-added mathematics data to be average, and one suggested to be weak, were not noticeably different in numeracy gains from three schools which were suggested by the LEA data to be above average, would suggest that our sample of schools contained mainly schools which were average to good in numeracy teaching in comparison with the overall population of schools. The only exceptions were one (independent) school, which appeared to be below average in gains, one (state) school that was very good and another where the performance was outstanding.

On the basis of this it would seem likely that, in relation to the national population of teachers, the *moderately effective* category of teachers were likely to be about average in numeracy effectiveness, with some below average and some slightly above. The *effective* teachers would seem likely to be good to very good, with the *highly effective* group showing outstanding performance. The labels for the categories were chosen to reflect this distribution.

Because of the difficulties demonstrated in this study in identifying the comparative effectiveness of both schools and teachers on the basis of current data available, no closer estimate of the nature of the sample can be made.

1.11 References for Chapter 1

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Chapter 2: A framework for beliefs, knowledge and practice

2.1 Introduction

In this chapter we set out a model of the interplay and relationship between beliefs, knowledge and classroom practices. Each, we suggest, informs and is informed by the others. Understanding why some teachers may be more effective than others requires an examination of each of these aspects. We examine these aspects in the following sections:

2.2 Beliefs, knowledge and practices

2.3 Aspects of beliefs and pedagogic content knowledge

2.4 Gathering information on beliefs and pedagogic content knowledge

The model we present here will be used as a framework for the analysis of the findings of the study in Chapters 3 and 4.

2.2 Beliefs, knowledge and practices

Our starting point for understanding effective teachers is a model of *teachers' classroom practices* informed by two complementary aspects: a set of *beliefs*, and a collection of knowledge (including subject knowledge) and understandings that teachers have about numeracy and its teaching which we refer to as *pedagogic content knowledge*. The model developed builds upon research previously carried out in teachers' beliefs and knowledge (see for example Aubrey, 1994; Bennett, Summers, & Askew, 1994; Lerman, 1990; Shulman, 1987; Thompson, 1984; Thompson, 1989)

We start with the assumption that *practice* in the classroom, in lessons, is the major factor influencing learning outcomes. Teachers' beliefs and knowledge and their practices outside the classroom, for example in their lesson planning, will all inform and influence lessons. However, it is the interactions between teachers and pupils as they occur in lessons that will be the most significant influence that a teacher has on pupils' learning.

We do not suggest that the interaction is only a one-way process from teacher to child. Teachers' perceptions of pupils' knowledge, understanding and behaviour in lessons will feed back and influence their own beliefs, knowledge and practices. The relationship is actually even more complex since our evidence to be presented in Chapter 3 and the evidence of previous studies leads us to suggest that the implicit *beliefs* or theories that teachers have, together with their *knowledge*, themselves influence the way that teachers interpret classroom events. For example, if a teacher believes that the major factor in learning mathematics is the rote memorisation of routines, then pupil errors are more likely to be interpreted as the result of pupil carelessness or lack of attention. On the other hand, a teacher who believes that pupils are trying to make sense of information may interpret errors as arising from a misunderstanding rather than carelessness.

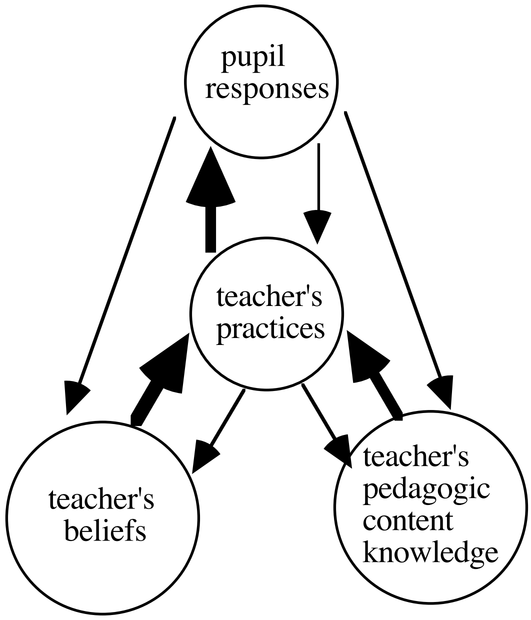

Figure 2.1 A model of the interplay and relationship between beliefs, knowledge and classroom practices

The arrows in the diagram also indicate that the relationship between teacher beliefs and teacher practices does not go only in one direction. For example, changes in practice that are driven by pragmatic reasons may lead to changes in beliefs. One of our teachers told us that she had started to get pupils to explain their methods to each other as it was an effective way of controlling a lively class. Through this practice she had come to realise the benefits of pupils articulating and sharing methods and now believed that it was an effective way of furthering the pupils' understanding. Another of the teachers talked about how her beliefs about pupils' abilities had been challenged through activities with pupils that she had been required to carry out on a CPD(continuing professional development) course.

Although the model does indicate this feedback loop, the use of bold arrows in the diagram indicates that we suggest that the strongest effect is likely to be that of teachers' implicit or explicit beliefs and pedagogic content knowledge shaping what happens in the classroom.

2.3 Aspects of beliefs and pedagogic content knowledge

We suggest that there are three aspects of beliefs that influence the teaching of numeracy:

- *Beliefs about what it is to be a numerate pupil.*
  This includes teachers' beliefs about:
  –the nature of mathematics in general
  –numeracy in particular
  –expectations of learning outcomes.
- *Beliefs about pupils and how they learn to become numerate.*
  Included here are beliefs about:
  –whether or not some pupils are naturally more mathematical
  –the type of experiences that best bring about learning
  –the role of the pupils in lessons.
- *Beliefs about how best to teach pupils to become numerate.*
  These are related to beliefs about teaching numeracy in terms of:
  –perception of the teacher's role in lessons
  –the influence of the 'accepted' wisdom of 'good' primary practice.

While a teacher's beliefs are important in shaping lessons, they are only part of the story. Different aspects of the teacher's knowledge also contribute in significant ways. A teacher might believe that pupils learn best through direct instruction. However, if a teacher's *subject knowledge* is limited or incorrect, what the pupils are likely to learn is going to be affected by this knowledge.

Similarly, if a teacher does not have good *knowledge of pupils* in terms of what they already know or how they, as individuals, approach tasks, the pupils may be expected to learn either something that they already know or something that is too complex for their current knowledge state.

Finally, while a teacher may have a sound understanding of a mathematical idea, suitable *teaching approaches* need to be used in order to make the idea accessible to pupils. For example, the mathematical concept of a fraction encompasses many different applications (part of a whole shape and part of a collection of items, to name but two). What is essentially a generalised abstract concept has to be presented to pupils through more concrete models (diagrams, physical materials, verbal analogies). Over time, pupils need to be introduced to a range of representations in order to understand the more abstract idea. For example, too much reliance on fractions being represented as congruent parts of a whole shape (e.g. three fifths as three pieces of a 'pizza' already cut into five sector pieces) may lead to a limited understanding of fractions and later difficulties in relating fraction notation to the idea of a ratio (e.g.

expressing one price as 7/5 of another) or of a rational number (representing 7/5 as a point on the number line between 1 and 1.5).

The types of representations used in lessons not only depends on a teacher's knowledge of teaching approaches but may also, in part, be determined by the teacher's belief orientation. For example a belief in the importance of being able to physically manipulate objects may lead to less attention being given to diagrams. But the greater a teachers' knowledge is in terms of awareness of different forms of representation the more likely it is that she will provide pupils with access to this range.

These three areas together contribute to what we are calling *pedagogic content knowledge*:

- *Numeracy subject knowledge*: understanding of mathematics and numeracy appropriate to what is being taught.
- *Knowledge of how pupils learn numeracy:* what particular pupils currently being taught understand as well as knowledge of pupils more generally, for example aspects of the numeracy curriculum that are generally found difficult, common misconceptions and models of progression.
- *Knowledge of numeracy teaching approaches:* understanding of teaching styles and different ways of presenting numeracy ideas to pupils, including a range of diagrammatic and verbal representations, so that they gain access to the subject knowledge.

This model of teacher beliefs and pedagogical content knowledge does not exist in isolation. The contexts in which teacher operate should also be taken into account. Contextual influences include:

- school practices and ethos
- the context of the National Curriculum and educational policies
- parental and pupil expectations.

But every study has to set boundaries and concentrate on the variables most relevant to its goals. For our study, the goal is knowledge about characteristics of teachers and factors that influence these so that others (the Teacher Training Agency, teacher trainers and teachers themselves) will benefit. Our attention to contextual influences will therefore be limited to those that help with this understanding.

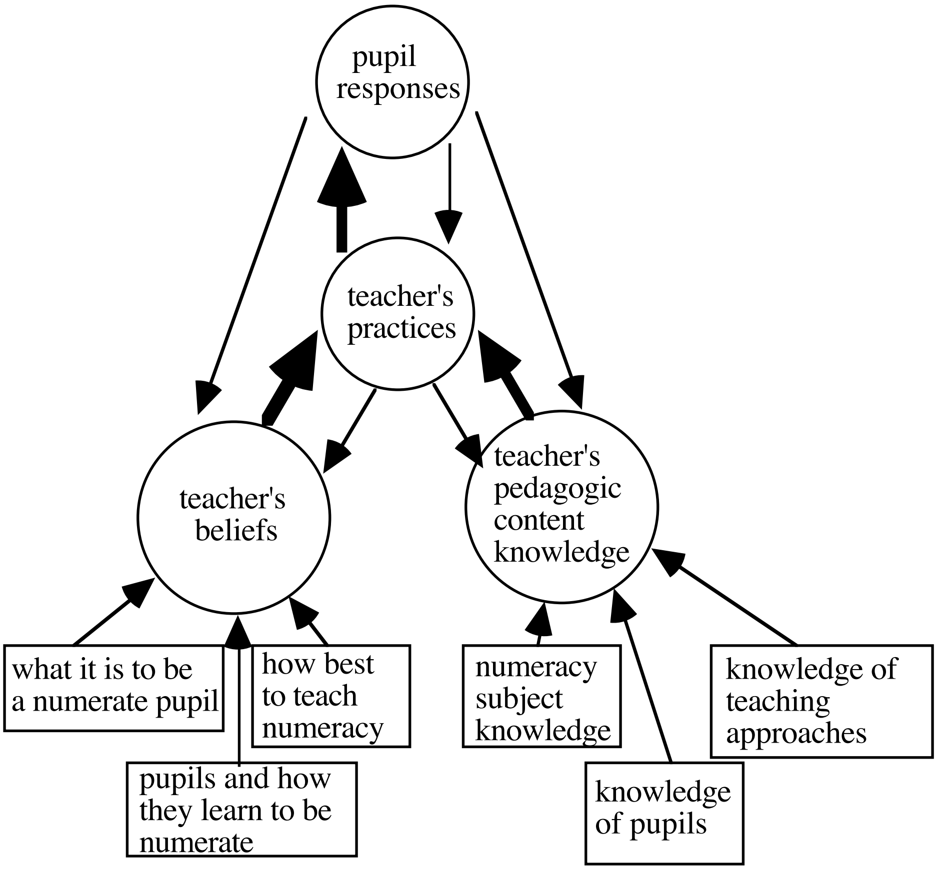

Figure 2.2 The elaborated model

2.4 Gathering information on beliefs and pedagogic content knowledge

In order to gather information on teachers' beliefs and pedagogic content knowledge, five methods were selected and developed to provide data, both qualitative and quantitative, on the teachers in the sample.

Questionnaire
A questionnaire was administered to all 90 teachers in both focus and validation schools.

This provided background data on:

- organisation and planning for mathematics teaching
- resources and classroom materials
- training and continuing professional development

and about teachers' perceptions of:

- teaching styles
- knowledge and beliefs regarding numeracy
- beliefs on teaching, learning and assessing mathematics in general.

Classroom observations
In total, 84 lessons were observed, three for each of the 18 case study teachers in focus schools and two for each of the 15 validation teachers. Data was gathered from observation in classrooms, including a focus on:

- organisational and management strategies - how time on task is maximised, catering for collective and individual needs, coping with the range of attainment
- teaching styles - intervention strategies, questioning styles, quality of explanations, assessment of attainment and understanding, handling pupil errors
- teaching resources - sources of activities, range of tasks, resources available, expected outcomes
- pupil responses - ways of working, evidence of understanding.

Detailed accounts were given of the flow, content and context of lessons. The attention that teachers paid to providing instruction in and discussion of more sophisticated strategies for calculation (as opposed to relying on counting strategies) was a focus of attention.

Head teacher interviews
Interviews were held with the 6 head teachers of the focus schools. These interviews probed issues arising out the refinement of teacher data, including:

- school policy, management and approaches to continuing professional development (CPD) as related to the research
- heads' perceptions of teachers' confidence, ability and approaches to teaching.

Case study teacher interviews
Fifty-four interviews with case study teachers in focus schools were carried out, three for each of the 18 teachers.

- background interview: this interview provided evidence to supplement the questionnaire on training and experience as well as information on beliefs, knowledge and practices in teaching numeracy; teachers' own perceptions of what has made them successful teachers of numeracy, and reasons for factors identified
- 'concept mapping' interview: this interview was based around a task that explored the teachers' understanding of aspects of mathematics related to teaching numeracy
- 'personal construct' interview: this interview was structured around a task that focused on the particular group of pupils that the teacher was currently teaching in order to explore the beliefs and knowledge about pupils and how they came to be numerate.

Validation teacher interviews
Each of the fifteen validation teachers was interviewed. These interviews were structured around a series of statements derived from the interviews with the case study teachers and selected to elicit responses to factors thought to be significant.

Appendix 2.1 provides further details on each of these sources of data.

The next chapter sets out our findings about *teacher beliefs* arising from this data, while Chapter 4 explores the *teachers' pedagogic content knowledge*.

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Chapter 3: Teachers' belief systems

3.1 Introduction

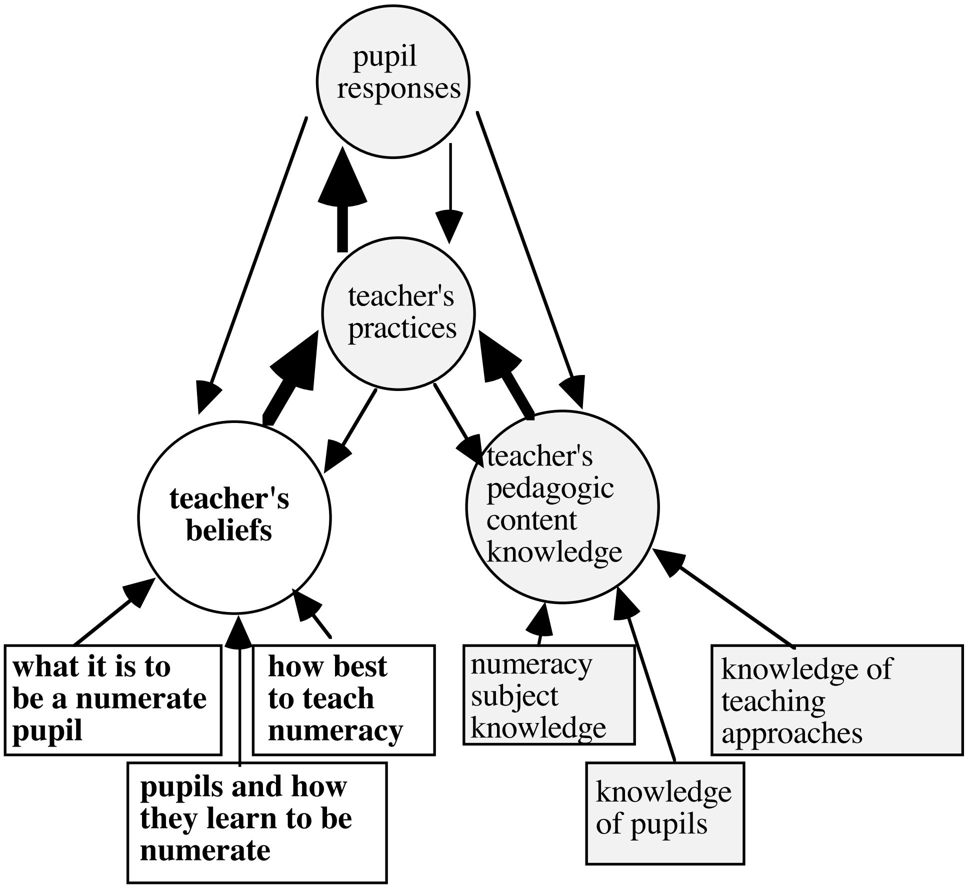


Figure 3.1 Focus on the part of the model concerned with a teacher's beliefs

In this chapter we focus on teachers' beliefs, and present three models of sets of beliefs that emerged as important in characterising, and helping to understand, the approaches teachers took towards the teaching of numeracy. We discuss these three models in terms of three orientations towards teaching mathematics:

- *connectionist*
- *transmission*
- *discovery*.

These orientations are "ideal types": no one teacher did, or is ever likely to, fit exactly within the framework of beliefs of any one of the three orientations; many combined several characteristics of two or more orientations.

However, it was clear that those teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations.

Such beliefs are often implicit and tend to shape rather than directly control behaviour. One use for these findings is for other teachers to examine the extent of match between their own personal belief system and those of the teachers in this study whose pupils made most learning gains.

Our examination of these orientations is dealt with in the following manner:

3.2 Orientations towards mathematics teaching

An understanding of the teachers' beliefs and practices was built up from data from three sources

- questionnaire data from the full sample of 90 teachers
- observations of 54 mathematics lessons with the 18 case study teachers and 30 lessons with the 15 validation teachers
- three interviews with each of the 18 case study teachers and two with each of the 15 validation teachers, in particular but not exclusively the 'background' interviews with each teacher, following observation of a lesson, which explored teachers' perceptions of and explanations for their practice, both in these lessons and more generally.

The transcripts of interviews with and field notes of observations of the 18 *case study* teachers were coded and analysed using a 'constant comparative' method, identifying similarities as well as differences in the teachers' belief systems and practices. The data pertaining to teachers with relatively high and relatively low gains was examined first, and later the analysis was extended to data for other teachers. Validity of the findings was assisted both by triangulation between the different data sources on each teacher, and by the fact that each transcript for the case study teachers was analysed and discussed by at least three members of the research team.

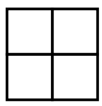
As a result of this analysis, we suggest that the similarities allow the beliefs to be characterised into three contrasting orientations towards the teaching of numeracy. These orientations were also apparent in the data from the 15 *validation* teachers, although the data for this group of teachers is less comprehensive.

Before setting out key differences in these orientations we offer three examples of classroom practice to provide some flavour of how each orientation may look in practice. The examples have been taken from field notes of actual lessons observed.

Example 1: Connectionist orientation
A Y6 class. The teacher has put a chart on the white board that has columns for fractions, decimal fractions, percentages and ratios. One value has been entered in each row and the pupils are working in pairs to convert from one form of representation to another. They are using a variety of methods but working mainly in their heads and most are checking using a different method. As they begin to complete the task the teacher brings the class together. Individuals are invited to provide the answers and explain the method of calculation used. The other pupils are attentive to these explanations. More efficient methods are offered and errors dealt with in a supportive manner either by the teacher or other pupils. Finally they discuss the sort of contexts where the different representations would be used.

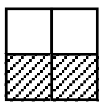
Example 2: Transmission orientation
A class of Y4 pupils is working on equivalent fractions. The teacher draws a diagram on the board to demonstrate a means of converting 1/2 into quarters. She explains that quarters are the fraction to convert to and so the pupils will need to draw a rectangle divided into four equal parts.

$$\frac{1}{2} = \frac{\square}{4}$$



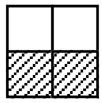
Since a half is required then two of these parts need to be shaded in.

$$\frac{1}{2} = \frac{\boxed{2}}{4}$$



'So, a half is equivalent to two quarters', explains the teacher. 'On the other hand', she continues, ' we could just look at the numbers on the bottom of the fraction. I have to multiply 2 (pointing to the 2 on the bottom of the 1/2) by 2 to make 4 (pointing to 4 on the board of a yet denominator free quarter fraction), so I multiply the 1 (pointing to the 1 on the top of the 1/2) by 2 also. So we get 2/4, which is the same as we got when we drew the diagram.'

$$\overset{\text{x 2}}{\underset{\text{x 2}}{\frac{1}{2} = \frac{\boxed{2}}{4}}}$$



The pupils are given a number of fractions to convert into equivalents and told they can either use the diagram method or the multiplication method. As the teacher moves around the class, once pupils have done a few examples using the diagram, she suggests to them that it will be quicker to use the other method.

Example 3: Discovery orientation
This Y2 class is organised in ability groups. The teacher is working with a low attaining group on doubling. The pupils spend a long time counting out individual cubes, fitting them together and recounting them. The teacher sets them a series of numbers to double, reiterates how to find the answer using the cubes and goes to join another group. The pupils are able to talk about what double four hundred might be and quickly move onto discussing doubling three thousand, six million and so on. After the lesson the teacher explained that she was concerned that the pupils were not ready to be working with large numbers, particularly as no Dienes blocks had been got out so that they could see that double three thousand was six thousand.

The following three sections (3.3, 3.4 and 3.5) describe characteristics of each of these orientations according to the three aspects of beliefs shown in the elaborated model in Figure 3.1. These are then summarised in a table presented in section 3.6.

3.3 The *connectionist* orientation

Connectionist beliefs about what it is to be a numerate pupil
We characterise a *connectionist* orientation as including the belief that being numerate involves being both efficient and effective. For example that while 2016 - 1999 can be effectively calculated using a paper and pencil algorithm, it is more efficient to work it out mentally. However, for calculating 2345 - 1767, a reliable paper and pencil method (possibly but not necessarily one of those traditionally taught) would be both effective and efficient for many pupils. Methods of calculation are not completely divorced from the size and type of numbers being operated upon and connections need to be established between numbers and methods. Being numerate, for the *connectionist* orientated teacher, requires an awareness of different methods of calculation and the ability to choose an *appropriate* method.

Connectionist orientated teachers also emphasise the links between different aspects of the mathematics curriculum. Again, *appropriateness* is important in appreciating these links, for example in working with pupils to help them decide whether a problem presented in the form of fractions is best solved as fractions or whether to convert these and work with decimals.

The application of number to new situations is important to the *connectionist* orientation: they encourage pupils to draw on their mathematical understandings to solve realistic problems. But *connectionist* teachers go beyond interpreting Using and Applying Mathematics (UAM) (as set out as an Attainment Target in the National Curriculum) as the inclusion in the curriculum of investigations or contextual problem solving. The connectionist orientation places a strong emphasis on developing reasoning and justification leading to the proof aspects of UAM. Reasoning about number is as important as its application, and as such UAM becomes integral to the teaching of number.

Connectionist beliefs about pupils and how they learn to become numerate
Associated with the *connectionist* orientation is a belief that most pupils are able to learn mathematics given appropriate teaching, which explicitly introduces the links between different aspects of mathematics.

Further to this is a belief that pupils come to lessons already in possession of mental strategies for calculating but that the teacher has a responsibility for intervening, working with the pupils on becoming more efficient. Any misunderstandings that pupils may display are seen as important parts of lessons, needing to be explicitly identified and worked with in order to improve understanding.

Connectionist beliefs about how best to teach pupils to become numerate
The primary belief here is that teaching mathematics is based on dialogue between teacher and pupils, so that teachers better understand the pupils' thinking and pupils' can gain access to the teachers' mathematical knowledge. This belief manifested itself in practice through extensive use of focused discussion to help pupils explore efficient strategies and interpret the meaning of mathematical problems.

Alongside this was a belief that some of the complexity of mathematical ideas had to be presented to pupils. For example, fractions, decimals and percentages were taught together, rather than as separate topics.

3.4 The *transmission* orientation

Transmission beliefs about what it is to be a numerate pupil
The transmission orientation entails a belief in the importance of a collection of procedures or routines, particularly in regard to paper and pencil methods, one for doing each particular type of calculation regardless of whether or not a different method would be more efficient in a particular case.

The transmission orientation encompasses a view of using and applying as the application of mathematics to word problems (basic calculations set in a real world context). These word problems can be tackled after learning to do calculations or procedures in an abstract form. Since the numeracy emphasis is on the ability to perform set routines, the transmission orientated teacher, at primary level, does not give much attention to the reasoning, logic and proof aspects of UAM.

Transmission beliefs about pupils and how they learn to become numerate
Since the transmission orientation places such emphasis on the ability to reproduce set methods and routines, what the pupil already knows is of less importance, unless it forms part of a new procedure. Methods that pupils have created themselves are not used as the basis from which to build more efficient and effective methods.
Pupils are believed to vary in their ability to become numerate; if the teaching has explained a method clearly and logically, then any failure to learn must be the result of the pupils' inability rather than a consequence of the teaching. Any misunderstandings that pupils may display are seen as the result of the pupils' failure to 'grasp' what was being taught; they need to be remedied by further reinforcement of the 'correct' method and more practice to help them remember.

Transmission beliefs about how best to teach pupils to become numerate
The *transmission* orientation places more emphasis on teaching than learning. Thus teaching is believed to be most effective when it consists of clear verbal explanations of routines. Interactions between teachers and pupils tend to be question and answer exchanges in order to check whether or not pupils can reproduce the routine or method being introduced to them.

Application problems tend to be 'word' problems - situations, often contrived, that are used as context for the further practice of calculating routines. Teaching is focused on the introduction of strategies for 'decoding' word problems to identify the operation they 'contain'.

Further, this emphasis on a number of routines and methods to be learned leads to the presentation of mathematics in discrete packages, for example, fractions are taught separately from division.

A *transmission* orientation also results in the use of 'track laying' - the use of paper and pencil methods and styles of recording in advance of when they may be appropriate. The use of vertical forms of recording encourages a concentration on operating with single digits, concentrating on one 'place' at a time (e.g. first dealing with the units, then the tens, and so on). These may well be introduced first with single digit and simple 2-digit numbers that would more efficiently be operated on mentally.

3.5 The *discovery* orientation

Discovery beliefs about what it is to be a numerate pupil
The discovery orientated teacher tends to treat all methods of calculation as equally acceptable. As long as an answer is obtained, whether or not the method is particularly effective or efficient is not perceived as important. Pupils' creation of their own methods is a valued process, and is based upon building up their confidence and ability in practical methods. Calculation methods are selected primarily on the basis of practically representing the operation. The mathematics curriculum is seen as being made up of mostly separate elements.

Using and applying mathematics is seen as primarily concerned with the use of practical equipment.

Discovery beliefs about how pupils learn to become numerate
The primary belief here is that becoming numerate is an individual activity derived from actions on objects. Learning takes precedence over teaching and the pace of learning is determined by the pupils. Pupils' own strategies are the most important: understanding is based on working things out for themselves.

Pupils are seen as needing to be 'ready' before they can learn certain mathematical ideas. This results in a view that pupils vary in their ability to become numerate. Pupil misunderstandings are the result of pupils not being 'ready' to learn the ideas.

Discovery beliefs about how best to teach pupils to become numerate
Teaching pupils requires extensive use of practical experiences that are seen as embodying mathematical ideas so that pupils discover methods for themselves. Mathematical ideas need to be introduced in discrete packages.

Learning about mathematical concepts precedes the ability to apply these concepts and application is introduced through practical problems.

3.6 Summary of characteristics of orientations

Table 3.1 sets out the key distinctions between these orientations that have been discussed in sections 3.3, 3.4 and 3.5.

It should be appreciated that the entries in the table are for purposes of clarity and brevity necessarily somewhat crude characterisations; they concentrate only on the major priorities held by teachers with different types of orientation. For example the first two entries under *transmission* refer to performing pencil and paper methods of a standard kind, and doing this with confidence. There is no suggestion that these objectives might not be shared by all types of teacher, but that only for *transmission* teachers are these the strongest priority. For *connectionist* teachers, for example, written methods are lower in priority than mental methods, and selecting a method that is efficient and effective in a particular problem is the most important priority of all.

	connectionist	**transmission**	**discovery**
Beliefs about what it is to be a numerate pupil	Being numerate involves: • the use of methods of calculation which are both efficient and effective; • confidence and ability in mental methods; • selecting a method of calculation on the basis of both the operation and the numbers involved; • awareness of the links between different aspects of the mathematics curriculum; • reasoning, justifying and, eventually, proving, results about number.	Being numerate involves: • primarily the ability to perform standard procedures or routines; • a heavy reliance on paper and pencil methods; • selecting a method of calculation primarily on the basis of the operation involved; • confidence in separate aspects of the mathematics curriculum; • able to 'decode' context problems to identify the particular routine or technique required.	Being numerate involves: • finding the answer to a calculation by any method; • a heavy reliance on practical methods; • selecting a method of calculation primarily on the basis of the operation involved; • confidence in separate aspects of the mathematics curriculum; • being able to use and apply mathematics using practical apparatus.
Beliefs about pupils and how they learn to become numerate	• Pupils become numerate through purposeful interpersonal activity based on interactions with others. • Pupils learn through being challenged and struggling to overcome difficulties. • Most pupils are able to become numerate. • Pupils have strategies for calculating but the teacher has responsibility for helping them refine their methods. • Pupil misunderstandings need to be recognised, made explicit and worked on.	• Pupils become numerate through individual activity based on following instructions. • Pupils learn through being introduced to one mathematical routine at a time and remembering it. • Pupils vary in their ability to become numerate. • Pupils' strategies for calculating are of little importance - they need to be taught standard procedures. • Pupil misunderstandings are the result of failure to 'grasp' what was being taught and need to be remedied by further reinforcement of the 'correct' method.	• Pupils become numerate through individual activity based on actions on objects. • Pupils need to be 'ready' before they can learn certain mathematical ideas. • Pupils vary in the rate at which their numeracy develops. • Pupils' own strategies are the most important: understanding is based on working things out yourself • Pupil misunderstandings are the result of pupils not being 'ready' to learn the ideas.

	connectionist	**transmission**	**discovery**
Beliefs about how best to teach pupils to become numerate	• Teaching and learning are seen as complementary. • Numeracy teaching is based on *dialogue* between teacher and pupils to explore understandings. • Learning about mathematical concepts and the ability to apply these concepts are learned alongside each other. • The connections between mathematical ideas need to be acknowledged in teaching. • Application is best approached through challenges that need to be reasoned about.	• Teaching is seen as separate from and having priority over learning. • Numeracy teaching is based on *verbal explanations* so that pupils understand teachers' methods. • Learning about mathematical concepts precedes the ability to apply these concepts • Mathematical ideas need to be introduced in discrete packages. • Application is best approached through 'word' problems: contexts for calculating routines.	• Learning is seen as separate from and having priority over teaching. • Numeracy teaching is based on *practical activities* so that pupils discover methods for themselves. • Learning about mathematical concepts precedes the ability to apply these concepts • Mathematical ideas need to be introduced in discrete packages. • Application is best approached through using practical equipment

Table 3.1: Key distinctions between connectionist, transmission and discovery orientations towards teaching numeracy.

The implications for teachers' practices of the distinctions made in Table 3.1 will be discussed in the remainder of this chapter.

3.7 Aspects of orientations in practice

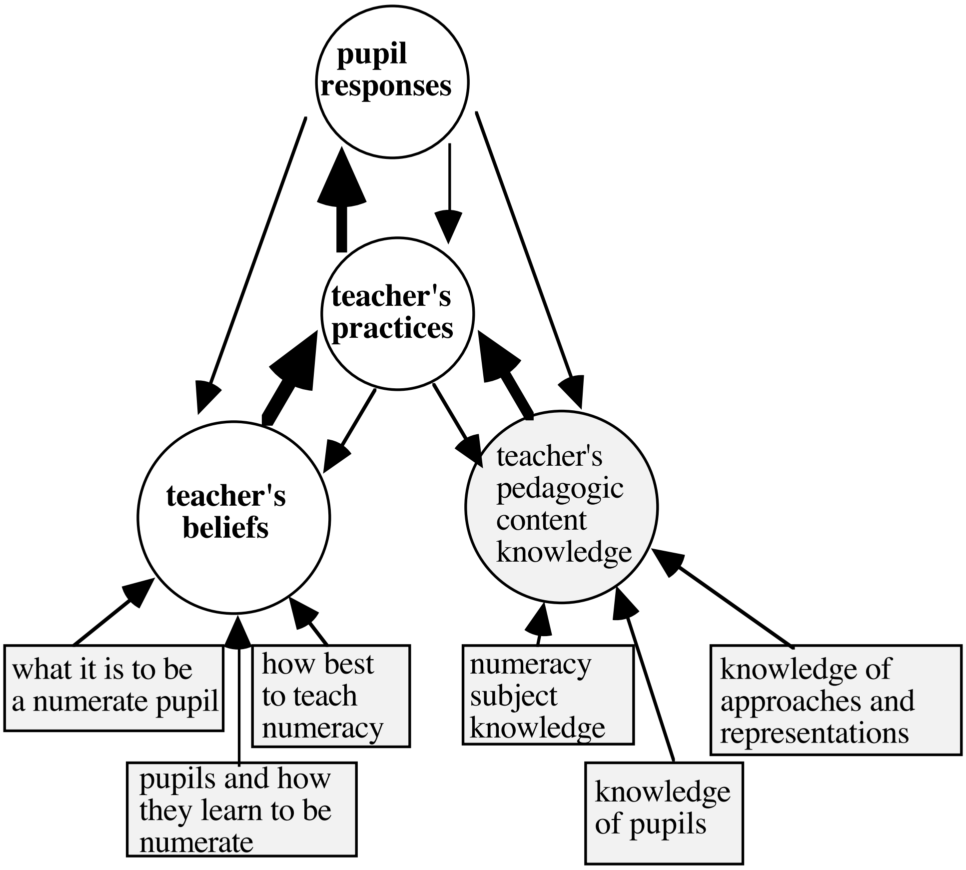


Figure 3.2 Focus on the part of the model concerned with the interplay between a teacher's beliefs and classroom practices

As previously indicated, the orientations of *connectionist*, *transmission* and *discovery* are ideal types: no single teacher is likely to hold a set of beliefs that precisely matches those set out within each orientation.

However, analysis of the data revealed that some teachers were more predisposed to talk and behave in ways that fitted with one orientation over the others. In particular, Anne, Alan, Barbara, Carole, Claire, Faith (the teacher initial matches the school code, so Anne and Alan are from same school), all displayed characteristics indicating a high level of orientation towards the *connectionist* view. On the other hand, Beth and David both displayed strong *discovery* orientations, while Elizabeth and Cath were both clearly characterised as *transmission* orientated teachers.

Other case study teachers displayed less distinct allegiance to one or other of the three orientations. They held sets of beliefs that drew in part from one or more of the orientations. For example, one teacher had strong *connectionist* beliefs about the nature of being a numerate pupil but in practice displayed a *transmission* orientation towards beliefs about how best to teach pupils to become numerate.

The connection between these three orientations and the classification of the teachers into having relatively high, medium or low mean class gain scores suggests that there may be a relationship between pupil learning outcomes and teacher orientations.

	Highly effective	**Effective**	**Moderately effective**
Strongly Connectionist	Anne Alan Barbara Carole Faith		
Strongly transmission			Cath Elizabeth
Strongly discovery			Beth David
No strong orientation	Alice	Danielle Dorothy Eva Fay	Brian Erica

Table 3.2. The relation between orientation and effectiveness

Year 1 teachers (Claire and Frances) are not included in the table since they could not be readily identified according to effectiveness on the basis of the testing occasion only.

In the following sections, quotations from the teachers are used to provide insight into the relationship between belief orientations and specific practices. As those teachers displaying a strong connectionist orientation were all in the highly effective category, their beliefs are particularly focused upon and contrasted with those of the strongly *transmission* and *discovery* orientated teachers.

3.8 Orientation and the role and nature of mental strategies in pupils becoming numerate

All the teachers encouraged pupils to have rapid recall of basic number facts. Over and above this, the connectionist orientated teachers also valued and worked on helping pupils develop a range of flexible and efficient mental strategies.

All the teachers, whether leaning towards a *connectionist*, *transmission* or *discovery* orientation saw some aspects of mental mathematics as important. Knowing basic number bonds and multiplication facts provided a baseline of expectations within all three orientations.

They must know their basic number bonds to ten and twenty. ... I think they must know their multiplication tables. ... They have a mental arithmetic test once a week. (Elizabeth Y5/6)

For Cath and Elizabeth, both *transmission* orientated teachers, being able to recall number bonds quickly was a skill that they worked hard to develop in the pupils. But calculations involving number bonds beyond 10 were expected to be done with paper and pencil. For Cath, knowing the number bonds was a means to an end, the end being able to move quickly onto paper and pencil calculations. Similarly Elizabeth had a primary aim of helping pupils work in the written mode.

We do a lot of work with them on adding up where you look in columns, and they've got to add up seven and three ... I do a lot of work with that so they can see that you can add up quickly if you know seven and three. (Cath Y4/5)

In the national curriculum there's a lot more of what I call Friday afternoon maths ... but they are not actually getting an awful lot down on paper. They are grasping concepts perhaps, but it is not going to help them through this exam, which of course is what we are here for. So I have got to get them to be able to write it as well. (Elizabeth Y5/6)

However, the *connectionist orientated* teachers viewed mental mathematics as going beyond this recall of number facts. Mental mathematics did not involve simply knowing number bonds but having a conscious awareness of connections and relationships to develop mental agility. In this quote, Barbara is referring to key items of numeracy knowledge that she identified in the concept mapping interview.

I think you've got to know that they are inverse operations those two (addition and subtraction), and that those two (multiplication and division) are linked, because when you are solving problems mentally you are all the time making links between multiplication, division, addition and subtraction. ... I think mental agility depends on you seeing relationships between numbers and being aware of links. (Barbara Y6)

This mental agility meant that for the *connectionist* teachers mental mathematics also involved the development of flexible mental strategies to handle efficiently number calculations. Working on mental strategies, they believed, laid foundations that extended the pupils' levels of competency. Developing confidence in flexible mental methods meant that pupils would be able to tackle calculations for which methods had not been taught.

If by the end of the year I can equip them with the ability to mentally be able to understand number, to use it, ... they will be able to do all sorts of things. My last year, at the end of my Year 4, I gave them a calculator game to do mentally and it was long division and they could all play the calculator *game and ... they all worked it out in their head. So they could do long division mentally. I hadn't taught them the mechanics but because they had so many strategies and basic skills they were actually able to do that. (Anne Y2/3/4)*

The connectionist teachers believed that the pupils should be encouraged to select certain methods in preference to others on the basis of efficiency. In order to do this, they used a variety of methods to raise pupils' awareness of mental strategies, including both building on pupils' explanations of their methods and explicitly teaching some strategies. Through well managed and focused discussion, a variety of strategies were explored and examined for their effectiveness.

We do discuss, and of course we don't just get one strategy, you get totally different ones, whichever is right for the child or whichever is quickest. They tend to go for the quickest method. (Barbara Y6)

The *connectionist* teachers also emphasised the importance of estimation.

If you've got a good ability to estimate and to know what a sensible answer is then you're very quick to pick up if something doesn't sound right. And if you can estimate you get very quick on mental arithmetic and you get very quick on oral skills. If you are good on those, you tend to be good on the other things. (Claire Y1)

(Claire was not technically ranked as 'highly effective' as she taught Year 1, but her pupils nevertheless did well in the April administration).

But the *connectionist* teachers went beyond simply eliciting methods from pupils and promoting efficiency. When appropriate, they taught particular methods to work out calculations mentally. The following excerpts are from Y3 and Y2 lessons.

Anne reminds the children that before half term they will all learn to add in their heads ..... 8, 7 and 11.
"Today we are going to learn to add 11. If we think of a number first, take 51 and add 11, can anyone do it?" Hands go up.
Boy "62"
Anne asks him how he did it.
Boy " because I knew it was 62".
Anne: "well if you add 10 to 51 you have 61 and add 1 unit so you add 1 to 61 so you have 62".
Anne: "87 add 11". Hands go up.
Anne: "add 10 to 80, add one to 7". (Anne, Y3)

Carole asks the children if anyone can add nine to 36 without using their fingers. She asks a child to explain how they knew the answer was 45.
"Well, I knew that 36 add 10 is 46 and I took off one."

> *The teacher reiterates the method and another child says that it can be done by taking off one and then adding 10.*
> *The class practices a few more examples before Carole directs them to a page in the text book that has a set of 'add nine' calculations. (Carole, Y2)*

Anne sums up the *connectionist* position.

> *I have tried to provide them with a whole range of different ways of going about adding numbers, or taking them away, so that they will be able to become comfortable with the strategies that they like best. ... Some teachers I know, their mental calculation is to walk around the classroom with, you know, eight and six are ... I am not trying to pressurise them, I am trying to equip them with the skills. (Anne Y2/3/4)*

In contrast, rather than explicitly exploring mental methods of calculation, Cath (*transmission* orientated) believed that learning tricks to solve problems easily was valuable as this freed them from having to think mentally.

> *Multiplying by ten they know they are moving figures backwards and forwards rather than having to do the mental arithmetic. (Cath Y4/5)*

Beth, a *discovery* orientated teacher, talked about the importance of mental methods but there was little evidence in her classroom practice of explicitly working on methods with the pupils. Instead, there was a sense of pupils having to come to an understanding only through their own efforts.

> *I did not actually ask how they did it ... they have got to do what comes into their head and what is the best way for them to do it.(Beth Y3)*

The *connectionist* teachers had high expectations of the pupils in terms of mental mathematics and, although they acknowledged that some pupils might be more talented mathematically, they expected most pupils to be able to become confident and competent in mental number.

Other teachers did not share this view, instead seeing mental mathematics as something only accessible to the more able pupils, as typified by this comment by a *discovery* orientated teacher:

> *What do I mean by higher ability? Mental work. Retention of facts- mental mathematics things like tables to ten and a good memory for that (David Y5).*

The attention paid to explicitly exploring pupils' mental methods was not just of benefit to the pupils. The *connectionist* teachers also indicated how much they learnt from the diagnostic assessment opportunities that it provided.

With that group this morning, I've actually found one child in that group is particularly good at mental arithmetic. ... Your view of a child is often limited by what they can do in a sum, a sum on a page and you can mistakenly think 'they've got all their sums right, they can do it'. But then you try to ask them to apply it and they can't apply it. (Barbara Y6)

While Cath (*transmission* orientated) would get pupils to explain their methods of working, this was only used when there was evidence that a pupil had failed to understand something.

I will have the lower group and go through things again, give them more ... help and more ... one to one ... and go through the things that they are finding difficult. (I) try to find out where it is that they are not understanding or going wrong. (Cath Y4/5)

By getting the pupils to share and explain their methods of solution whether or not they had got the right answer to a calculation, the *connectionist* teachers built up a detailed picture of pupils' strengths and weaknesses. This, in turn, enabled the teachers to plan lessons that challenged pupils' current levels of thinking, as typified by Alan, a *connectionist* orientated teacher.

Alan planned lessons to build upon and extend pupils' previous knowledge. Pupils were presented with situations where they were expected take the initiative and use their knowledge of what they had learned previously and apply it in a realistic context. Alan did not hesitate in providing pupils with challenges that they might not succeed at. Even if they got things wrong, there was a payoff in terms of further insight for the teacher into the pupils' understanding. Alan had set up a challenge where pupils had to interpret two pie charts. Two different sample sizes were used, 80 and 100, and pupils were challenged to distinguish between what could be said about preferences in the total population in each case as compared to the actual number of people responding in the sample. For example, some pupils argued that because 40 people in each sample had chosen computers as a favourite pastime then this was equally popular in each population.

I wouldn’t have expected them to have a particular problem with interpreting pie charts ... but (what) was novel was that there were percentages on the pie chart. ... It is evident from what they did this morning that although they now recognise percentages as a way of expressing a proportion, they haven’t really taken on board the practical use of percentages. ... So that is a piece of information that I have got that I didn’t have this morning. Also I think that the other thing that I have got out of it which interests me is that again, although they are very comfortable with the idea of a pie chart, they haven’t really got the idea that a pie chart is actually a precise thing as well as just giving them an impression. (Alan Y5/6)

Particularly noteworthy here is the extent to which Alan is talking in terms of understanding deep underlying mathematical structures rather than the surface level of whether or not they did the correct calculations. His use of a statistical context to probe and deepen understanding of percentages as well as of pie charts illustrates his belief in the importance of connecting different areas of mathematics.

3.9 Orientation and teacher orientations

The connectionist orientated teachers placed strong emphasis on challenging all pupils. In contrast the transmission and discovery orientated teachers may provide challenge for the higher attaining pupils but structured the mathematics curriculum differently for lower attaining pupils.

Among the *connectionist orientated* teachers there was a clear indication that they believed that pupils of all levels of attainment had to be challenged in mathematics. Being stretched was not something that was restricted to the more capable pupils.

> *I think high expectations, but not unreasonable expectations. ... Giving (the pupils) a challenge so (they) really have to stretch (themselves) and think about it but not something that was totally out of (their) capabilities. That would de-motivate. (Claire Y1)*

Anne believed that all pupils, even the youngest, enjoyed being given challenging tasks and in her experience would become totally involved in problem solving activities.

> *They (a Y1 class she was covering)... had to choose three stars out of the whole display that I put together and they added them and we checked them along the number lines. ... Then they had a challenge which was to find the largest total, it could be three stars, four stars, five stars ... If you had seen them, every child was totally involved with what they are doing. ... Even the smallest children, you know, the children who have less confidence and feel less happy with number managed, ... so they were having the same experiences and a proper range ... Sometimes when they have had a chance to go away ... they can internalise what they have been learning and you come back and just find that they fly through things ... It is amazing how much even those you think have a problem, how much they do actually take part. (Anne Y2/3/4)*

The *connectionist* teachers indicated a commitment not to label pupils as being inevitably poor at mathematics. They had high levels of expectations for all pupils irrespective of ability. Intelligence was not seen as static and all pupils were regarded as having the potential to succeed.

But I have the same expectations for the children, I always think about it as not so much what the children are doing as what they have the potential to do. So even if I have children like Mary in the classroom who are tremendously able, I am really just as excited with the children who are having that nice slow start, because, who knows, tomorrow they may fly– you just don't know.(Anne Y2/3/4)

A view of working within levels that would not challenge pupils was held by both *transmission* and *discovery* orientated teachers and appeared to have at least two effects. First, lower attaining pupils in particular would seem to require a different approach to teaching and learning. There was little sense that these pupils were expected to achieve a sense of satisfaction through being challenged. Beth (*discovery* orientated) and Elizabeth (*transmission* orientated) both seemed wary of challenging the pupils at all:

Children should never be asked to do things that you do not think that they can do ... problems that they have to think about but that they know how to do.(Beth Y3)

I do think they need to feel ..in every maths lesson, they've either understood something or they've done something well. (Elizabeth Y5/6)

Second, in order to reduce the demands on pupils, the mathematics had to be presented in small, fragmented steps. Because this breaking down and structuring had to be done by the teacher, it appeared that this in turn fostered a classroom culture where some pupils became heavily dependent upon the teacher and a style of learning mathematics characterised by lack of deep understanding. Thus a cycle of further dependency and low attainment may be set up. Elizabeth (*transmission* orientated) and David (*discovery* orientated) explained how they deal with pupils having difficulties.

A lot of them learn by rote... one who needs extra help, I will stand behind him when he is doing it and actually working with him for a long long time ... so they (the weakest) get a lot more of my help. Minus minus something, really I teach it by rote. ... These two are very weak ... They have to learn it by rote...if the child gets a low mark it's probably my fault not the child's. ... No child ever fails. (Elizabeth Y5/6)

They have to have things explained. They are both struggling, struggling, struggling mathematicians, they need extra input, need extra input. As much extra help as possible, encouragement, breaking down their skills to smaller particles. (David Y5)

Both Elizabeth and David in being helpful appear to structure task so that pupils achieve short term success, either by rote learning or only having to deal with a small part of a mathematical task. As indicated above, this may lead to pupils not appreciating the 'broader' mathematical picture and not being able to move on.

This breaking the subject down into small parts stands in contrast to the *connectionist* orientated teachers' commitment to presenting a more coherent view of the subject. Alan was typical in this respect in emphasising the ability to see the connections between different areas of mathematics. This was reflected in a teaching style that made the links between different areas of mathematics explicit. Part of this involved presenting pupils with situations in which they had to make use of the ideas they were being taught.

> *The relationship between writing something as a fraction and it being the equivalent of what it is written in percent and also the equivalent to what it is written as a decimal. ... I think that having the children recognise this sort of equivalence is something that I think is important to them being numerate at an early age, because we often teach them the arithmetic of those and it is often taught in isolation and it is taught as things again that aren't necessarily connected. Here is a way to add fractions, here is a way to add decimals. One might be taught at the beginning of the term, one might be taught at the end of the term and there isn't necessarily any recognition given that actually the two things can be used to check one another, they can be used one in place of another and that they can be taught at the same time. (Alan Y5/6)*

Throughout the lessons the pupils in Alan's class were frequently reminded to think about the relationships between the areas of mathematics, which he wanted them to focus on, as this extract from the classroom observations illustrates.

> *Pupils come in and are directed to work on percentages written on the board, which they are to do in the first 10 minutes. The teacher reminds the children "to keep in their head all the time that there is a relationship between ..... fractions, decimals, fractions and percentages, keep that in your mind" and to look at work done previously in their exercise books if they need to. Teacher asks how many percent is 4/100. hands up. G "4%". Teacher crosses out fraction and writes the percentage. He asks how many percent is 4/10. hands up. B "40%". Teacher crosses out fraction and writes the percentage. Asks if the pupils can see they relationship, "4%, 40%, 10 times bigger". Teacher asks how many percent is 3/5 and reminds the pupils to think about equivalent fractions. Hands up. G "60%". (Alan Y5/6)*

Alan continued to stress the links between decimals, fractions and percentages throughout the lesson. Each time an answer was given Alan asked the pupil to explain how they got that answer rather than just accepting the answer as given.

3.10 Orientation and style of interaction

As well as carefully selecting tasks, pace was an element in keeping pupils focused and challenged. Anne encouraged pupils to work speedily through one activity in order to move on to the next one.

I mean ...children that I have work very hard and they work consistently. ... Whatever I am doing I try to keep the pace up as well. You know, I think that is terribly important, I think they seem a lot quicker. (Anne Y2/3/4)

The *connectionist* teachers' lessons were generally characterised by a high degree of focused discussion between teacher and whole class, teacher and groups of pupils, teacher and individual pupils and between pupils themselves. The teachers displayed the skills necessary to manage effectively these discussions. The teachers kept pupils focused and on task by organising these discussions around problems to solve, or sharing methods of carrying out calculations. For example, Claire's Y1 class had been modelling numbers to 100 by putting cubes in two hoops to represent tens and ones respectively, and then recording and reading the numbers on a hundred square. She asked the pupils how the square might be continued. One pupil suggested adding a third hoop and a lively discussion ensued as to where it should be placed in relation to the other two hoops and the order in which the digits should be recorded.

In school A, one of the *most effective* schools, there was a consistent approach to interacting with pupils throughout the years. Right from KS1 pupils were expected to be able to explain their thinking processes. Because the pupils were explaining, rather than simply providing answers to questions that the teacher already knew the answer to, the lessons were characterised by dialogue. In this discussion both parties, teacher and pupils, were having to listen carefully to what was being said by others. The result was pupils who, by Y6, were confident and practised in sharing their thinking and challenging the assumptions of others.

If I am honest with myself I probably spend more time talking with them than doing exercises and things like that ... because I want them to be able not to just give an answer, I want them to explain the process and what they are doing, to be looking for these links again, and to be able to be adventurous as well, because I think that part of numeracy is being prepared to stick your neck out and say 'I think this works because', and then to bounce it off the other people in the group and with everybody else not to just say 'oh no you are wrong', but to say 'no you wrong because', and then to perhaps find that the person who interrupted was wrong because there is another way of doing it. (Alan Y5/6)

As noted earlier, getting pupils to talk about their work in a frank and honest way also gave valuable insights into pupils' level of understanding and highlighted the areas where individuals might be encountering problems.

I can't work with the child unless I am able to have some toe hold as to what the child's strengths and weaknesses are. I can test a child, I can, in a formal setting, but I find it so important to be able to communicate with the child on a one-to-one level and to have the child be open and honest with me. (Alan Y5/6)

The *discovery* or *transmission orientated* teachers tended to talk about discussion either as a means of engaging pupils' interest or to provide the opportunity for pupils to share finished results. Elizabeth talked about how she brought pupils out to work on the board but only in the context of continuing a long division calculation she had started herself in order to keep the other pupils' interest. Beth, while seeing explanations as providing some means of linking ideas, also gave priority to the motivational aspect.

> *They are very proud of what they do, they like to show everyone and explain how they have worked things out. (Beth Y3)*

The key issue here is that at one level, the pupils were all getting a similar experience–the opportunity to describe their thinking. However what the teachers listened for and picked up on was dependent upon their orientation towards the mathematics and was subtly different across the three orientations, resulting in different outcomes for the pupils.

The *connectionist* orientated teachers worked more actively with the pupils' explanations, accepting what the pupils could do but also worked on refining them and drawing pupils' attention to differences between methods, raising questions of efficiency.

The *transmission* orientated teachers listened for how well the pupil's explanation matched theirs. Any errors or alternatives were simply corrected rather than made explicit and used as a means of furthering the pupils' understandings.

The *discovery* orientated teachers were more interested in the range of methods that the pupils produced, accepting them all as a way of valuing the pupils' powers of invention.

3.11 Orientation and the role of using and applying

All the teachers, whatever their orientation, acknowledged the importance of pupils being able to apply their computational skills to real-life problems. However, there was a marked difference between the *connectionist* teachers and either the *transmission* or *discovery* orientated teachers in the way that they characterised the role of Using and Applying Mathematics (UAM), a national curriculum attainment target.

For Beth (*discovery* orientated), application of knowledge involved pupils putting what they had previously learnt into context. Problems present 'puzzles' where the pupils already have the required knowledge and the challenge is only to sort out which bit to use. Alternatively, problems are a means of demonstrating to pupils the value of what they are learning.

If mathematics was to be meaningful and be seen to have relevance to everyday life, it had to presented as far as possible in a realistic context and not be just a book based activity. (Beth Y3)

Elizabeth (*transmission* orientation) clearly showed how she viewed UAM as something that happened after the mathematical content had been taught.

Yes, we also do the four rules of money for that. After they've studied that, then we can connect it to problems. (Elizabeth, Y5/6)

Reasoning, for Elizabeth was mainly to do with the ability to translate word problems into number operations.

(Interviewer: Can you talk to me about reasoning and what you mean by that?)
Well the sort of ability to read something in words and translate it into a basic problem. ... We work a lot on the words, you know, what's more than, what's less than ... so they get the idea of what the different words stand for. (Elizabeth, Y5/6)

The *connectionist* orientated teachers also recognised the importance of being able to apply computational skills. But over and above this they did not see it as a necessary pre-requisite that pupils should have learnt a skill in advance of being able to apply it. Indeed, the challenge of an application could result in learning. Claire, a strongly *connectionist* orientated teacher, taught a Y1 class. She believed that pupils benefited from sometimes being presented with application situations for which they did not always have the skills immediately available.

If they can't apply it directly then, that is where they would have the confidence (to) look at all the different approaches and then maybe start solving problems. ... It is not always a good thing to always be able to do something, it is not always a good idea if you can always do everything because there is no challenge there then. If you are not always able to do it that is when you start thinking in a different approach and broadening your base. (Claire Y1)

Giving children the opportunity to use and apply their knowledge in a mathematical context was integral to Anne's teaching. Opportunities were provided for children to demonstrate their understanding by initiating their own ideas, designing and planning their own work. But perhaps more importantly, children were encouraged to demonstrate their understanding by justifying their reasoning to others.

If they can use the language of mathematics to explain the patterns; to explain why they have chosen a certain method in multiplication. ... If they can talk about multiples and factors and square numbers and square roots - if they can talk about those things they have an understanding, knowledge of *them. If they can apply them then they will be able to develop onto being able to plan and completing an investigation. ... Last year I had seven year olds who could compute and subtract thousands ... and I'll say you can design three rules of your own. ... I say don't forget negative numbers. (Anne Y2/3/4)*

Barbara, one of the most effective *connectionist* orientated teachers had studied A-level mathematics but did not gain the qualification. In contrast to her own experiences, she now saw mathematical application as central to the learning of mathematical concepts.

I did maths for A Level. ... But actually I hadn't really sorted out how number worked until it was pointed out to me as a student how things actually worked. ... I'd always performed tricks until it was pointed out what the sums actually meant. ... I was sold on the idea they really do need to know to be able to APPLY it, they really need to know what they're doing with number. ... We have lots of using and applying ... and I think that has got to be done alongside teaching new concepts. ... It should be built into the maths programme, not separate from it, not 'let's do using and applying this week'. (Barbara Y6)

On the other hand, Beth (*discovery* orientated) acknowledged the importance of UAM but saw it as only able to be implemented once children had learned the basics.

I will give them just some basic sums set out ready for them similar to the ones we've done today and then problems where they have got to actually extract the information and use what we were doing today ... The more able children have done problem solving I know, but there are so many basics they have to cover that it is only as you get further up the school that more opens up to you really I think. (Beth Y3)

Similarly Cath (*transmission* orientated) felt that the mathematical 'content' had to take priority over UAM

We can't do as much of this investigation with various resources as perhaps the national curriculum would like us to do at the level I teach. ... I haven't got time. (Cath Y5/6)

While David (*discovery* orientated) thought that concepts could be learnt through UAM this was a fall back position rather than an initial approach.

Problem solving activities ... sometimes it's a way through to the children at the end if you can find practical problems, investigation (David Y5)

3.12 Discussion and implications of orientations

The importance of these orientations lies in how practices, while appearing similar may have different purposes and outcomes depending upon differences in intentions behind these practices. As other research on developing teaching has demonstrated, exhorting teachers to adopt particular practices without a deep understanding of the principles behind these practices does not in itself lead to raised standards (Alexander, 1992).

Equally, we would suggest that these orientations towards teaching mathematics need to be explicitly examined in order to understand why practices that have surface similarities may result in different learner outcomes. For example, our highly effective teachers demonstrated a range of classroom organisation styles including whole class teaching, individual and group work. On such measures their practices were indistinguishable from those of the teachers who were only moderately effective. While the interplay between beliefs and practices is complex, these orientations provide some insight into the mathematical and pedagogical purposes behind particular classroom practices and may be more important than the practices themselves in determining effectiveness.

Other teachers may find it helpful to examine their belief systems and think about where they stand in relation to these three orientations. In a sense the *connectionist* approach is not a complete contrast to the other two but embodies the best of both of them in its acknowledgement of the role of both the teacher and the pupils in lessons. Teachers may therefore need to address different issues according to their beliefs: the transmission orientated teacher may want to consider the attention given to pupil understandings, while the discovery orientated teacher may need to examine beliefs about the role of the teacher.

References for chapter 3

Alexander, R (1992). *Policy and Practice in Primary Education*. London: Routledge.

Chapter 4: Teachers' pedagogic content knowledge

4.1 Introduction

This chapter describes our answers to the question as to whether and how the various aspects of teachers' pedagogic content knowledge affect pupils' learning and attainment.

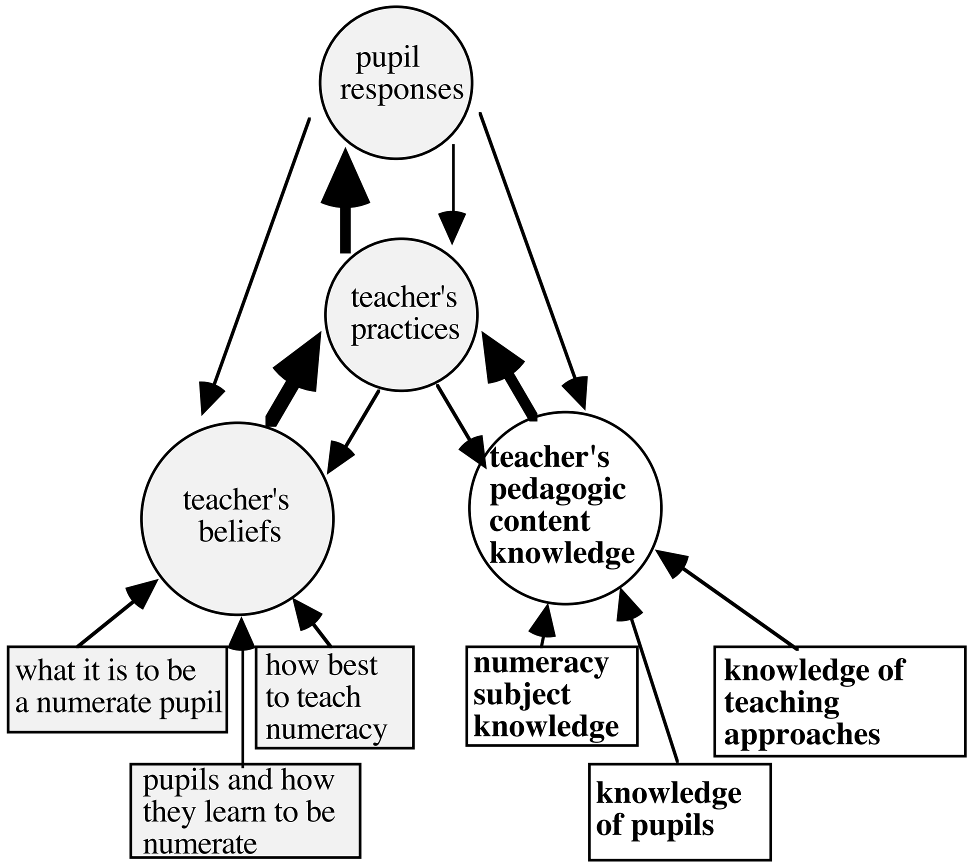


Figure 4.1 Focus on the part of the model concerned with a teacher's pedagogic content knowledge

Teachers' pedagogic content knowledge can be split initially into three aspects:

- *numeracy subject knowledge:*
    - how much do teachers know about numeracy as a whole?
    - what specific numeracy content knowledge do teachers have?
- *knowledge of pupils:*
    - what do teachers know about pupils' knowledge of numeracy?
    - how do teachers assess and record pupils' numeracy knowledge?
- *knowledge of teaching approaches*.

The third aspect, *knowledge of teaching approaches* will be discussed briefly first. in section 4.2. Then the first two aspects will be dealt with in order and in greater detail in sections 4.3 through 4.14.

4.2 Knowledge of teaching approaches

This aspect has already been largely dealt with in Chapter 3, when teachers' practices were described under the three different orientations identified (*connectionist*, *discovery* and *transmission*). Only a quick summary will therefore be provided here.

It was clear in particular that:

- *connectionist* teachers, who were highly effective, concentrated on assisting pupils to develop efficient conceptually based strategies, and in doing this used discussion and challenge to introduce links between different meanings and representations
- *transmission* teachers, who were only moderately effective, emphasised pupils' acquisition of a set of standard methods for solving a limited range of routine problems, by demonstrating specific methods and ensuring that pupils practised them
- *discovery* teachers, who were only moderately effective, emphasised pupils' own development of concepts and strategies, using practical activities and experience provided by the teacher as a basis.

This meant that teachers of each orientation tended to have knowledge of different types of approach, and although each might use the same representation (e.g. a number line) it was used for different purposes. In general, to make a crude characterisation:

- *connectionist* teachers generally had a wider knowledge including both practical and formal methods and representations and also a knowledge of informal strategies that different pupils had found useful
- *transmission* teachers seemed to keep to a narrow range of written methods and representations, using pictorial representations for an initial introduction and justification before moving swiftly to a formal method
- *discovery* teachers had knowledge of practical equipment which would assist pupils to obtain answers, and of standard formal methods, but were not always clear how best to help children to bridge between them.

Some examples in Chapter 3, and others quoted in the sections on numeracy subject knowledge, which follow, illustrate these differences.

4.3 Numeracy subject knowledge

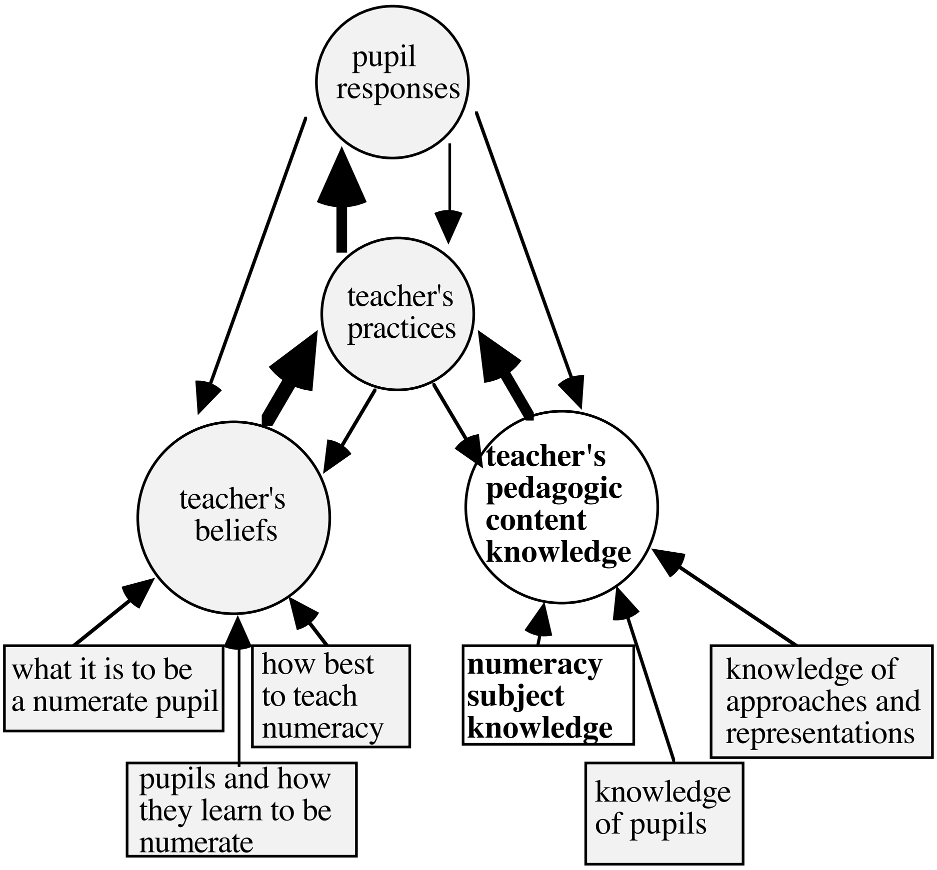


Figure 4.2 Focus on the part of the model concerned with a teacher's numeracy subject knowledge

A starting point for the project, built into the framework, was the assumption that a teacher's own subject knowledge would be an important aspect of their competence in teaching numeracy. Exactly what aspects of a teachers' knowledge made a significant difference in terms of pupil gains was much harder to identify than was anticipated. Certainly it turned out to be nothing as straightforward as the level of qualifications, or the fluency with which teachers could talk about ideas that contributed to numeracy.

- In terms of adequate understanding of mathematical concepts there was little to distinguish between the teachers.
- Some teachers were uncertain in regard to specific items of numeracy knowledge, but this was either at levels they were not teaching or in non-fundamental areas; either way there was little evidence that this would do clear damage to children's numeracy standards.
- *Connectionist* teachers tended to demonstrate a greater inclination to elaborate on the links between different numeracy concepts than did other teachers.

- *Transmission* teachers described proportionately more superficial techniques linking different numeracy ideas and fewer conceptual relationships between them than did other teachers.
- *Discovery* teachers and some of those not easily identifiable with one or other orientation, were little different in the depth of their responses from most *connectionist* teachers, but found it harder to elaborate on their knowledge.
- Teachers teaching younger age groups did not necessarily have a very deep understanding of concepts taught in later years, and in some cases confessed to panic when asked about them.

More details about these findings, with examples to illustrate them, are provided in sections 4.4 to 4.9 under the following headings:

4.4 Knowledge of content and relationships

4.5 Numeracy subject knowledge and teaching orientation

4.6 Numeracy subject knowledge and key stage

4.7 Teachers' mathematical qualifications

4.8 Correctness, adequacy and excellence of subject knowledge

4.9 Discussion and implications

An understanding of the teachers' numeracy subject knowledge was built up from data from three sources

- questionnaire data from the full sample of 90 teachers
- profiles of mathematical subject knowledge for the 18 case study teachers from focus schools, arising from the interviews, mainly but not entirely from the concept mapping interview
- observations of 84 mathematics lessons with the 18 case study teachers and the 15 validation teachers.

In general, analysis of the interview and observation data provided an understanding of teachers' strengths and weaknesses in particular aspects of numeracy and their association with pupil gains. Analysis of the questionnaire data allowed a more general picture to be developed of the relationship between level of formal qualification in mathematics and pupil gains.

It was not easy to decide how to elicit from our case study teachers in focus schools their knowledge of numeracy content. It did not seem either appropriate or helpful to give teachers a 'test' on numeracy; since what we wanted to access was less their formal ('decontextualised') knowledge than their 'craft' ('contextualised') knowledge i.e. how they were able to deploy content knowledge in planning and in teaching numeracy. In any case we had a surrogate measure of formal mathematical standard in that we had access in the questionnaires to the level of mathematical qualifications of each teacher, although in some cases these qualifications had been gained some years ago.

We also felt that any test would have unfairly discriminated against the Key Stage 1 teachers; teachers of Year 6 would be much more familiar with the type of 'test item' normally used in numeracy. In fact we did slip into the middle of the interviews at the point that teachers were talking about links between fractions and decimals an informal question as to whether the teacher would know how to convert 1/7, for example, into a decimal. In some cases with teachers of younger children this caused a panic response that upset at least one of them for the remainder of the interview. This suggested that it would have been difficult not only to do adequate justice to teachers' complex knowledge in any superficial direct 'test', but also only too easy to set off a 'panic' reaction.

We already had considerable evidence from the classroom observation of teachers' numeracy knowledge as used in teaching, including interactions with pupils. Some aspects of this have already been included in Chapter 3. Observation provided information that was comparable between teachers in the sense that it related to what they were doing with pupils. Nevertheless we thought that it would be valuable to have some data on teachers' more global understanding and knowledge of numeracy that was consistent in covering the same areas with all teachers. We therefore decided that the most appropriate method would be an interview that would allow teachers to talk informally about how they understood numeracy. This was called the 'concept mapping' interview, which was referred to briefly in Section 2.4.

During the concept mapping interviews, the 18 case-study teachers from the focus schools were asked to propose mathematical ideas that they considered to be important in numeracy (e.g. fractions, multiplication, estimating areas). They were then asked to show on a diagram how these concepts (supplemented where necessary by some suggested by the researcher) linked together, and also to explain the nature of the links. (A fuller description is given in Appendix 2.1.)

4.4 Knowledge of content and relationships

Analytical framework
From the data analysis it emerged that two distinct aspects of numeracy subject knowledge needed to be given attention:

- **knowledge of content** - knowledge of facts, skills and concepts of the numeracy curriculum, for example, knowing what a median is and how to calculate it
- **knowledge of relationships** - knowledge of how different aspects of the mathematics content relate to each other, for example, the relationship between decimals and fractions.

From the list of ideas that the teachers produced and the way they grouped them together, two measures of knowledge of the content of mathematics related to the teaching of numeracy were developed:

- *fluency* - the number of valid numeracy concepts suggested (the range given by teachers varied between 12 and 22)
- *scope* - the breadth of the teacher's vision of numeracy, measured by the number of broad aspects of numeracy touched on, e.g. whether the concepts given covered aspects such as the meaning of operations, methods of calculation, estimation, measurement, etc. (teachers volunteered concepts from between 5 and 10 different aspects of numeracy).

The concept mapping interviews were also analysed for the different ways in which the teachers identified and explained relationships between aspects of numeracy subject knowledge. The categories used for this, listed below, are not mutually exclusive (for example a relationship identified could be counted within the *explanation* category and then again within depth if it had been explained in *conceptual* terms):

- *links* - the number of legitimate links proposed between concepts, for example merely indicating that there is a link between fractions and decimals (between 12 and 23 links were noted by different teachers)
- *explanation* - the percentage of links that were at least to some extent explained. For example Danielle (no strong orientation) stated that both decimals and fractions were 'just ways of demonstrating parts of a whole' but chose not to elaborate further (this is partially accurate as an explanation but is neither comprehensive nor is it a very helpful way of characterising a quantity like 13.58 kilograms, or a use related to comparisons, such as that one side of a picture is roughly 7/5 of the other)
- *depth* - the percentage of links which are explained in conceptual terms rather than being only procedural (rule-based). For example, 'parts of a whole' would be regarded as conceptual, as would Barbara's (*connectionist* orientated) response quoted in the next section; whereas Beth (*discovery* orientated), a recent mathematics graduate, gave only a procedural explanation:

  You have got your fraction, you can change it into a decimal...I do not know, I do not know actually, I suppose because in my mind they are linked together but I find it very hard to actually think how and why, and why I would need to know...(Beth, Y3)

- *understanding* - the percentage of links well-explained. In addition to noting whether or not a link was explained between two concepts, the nature of the explanation was examined. For example in describing the links between fractions and decimals Barbara did not try to give a general definition but mentioned a number of aspects which came to mind, choosing to describe how she made the links in the classroom. Her answer included:

a tenth is 0.1, you've got to understand what a tenth is first so you've got to do work on fractions first, halves and quarters, obviously, leading onto sevenths, eighths, ninths and tenths ... .and they picture in their mind that .1 is one tenth ... .it's a shaded bit on the line of ten so they see its relevance to a whole, so .2 is two squares out of 10 ... .3.4 they'll think 3 whole sticks and .4 ... .fractions could be linked with proportion as well ... .how many so-and-so's will go into something ... .shape proportions like side to side measurement in art ... .you don't need fractions in everyday life very much, it's very hard to explain to the children that they need to know about fractions in a dividing sense and a sharing sense...with decimals of course there's a link with money...percentages of course are far more important than fractions.. (Barbara Y6)

Although a somewhat disjointed account, and lacking some key features, Barbara has expressed a number of important links, and also differences, between decimals and fractions. Fundamental aspects that she implied but did not quite explain, for example, include the observation that both fractions and decimals can be used to denote intermediate numbers between whole numbers on a number line such as that used for measurement scales (technically not all numbers since non-recurring decimals like the square root of 2 or π are not expressible in fraction form). However it is customary only to use halves and quarters for measurement and generally to employ decimals. She might also have added that any fraction could be expressed as a decimal, and that decimals can be used as well as fractions in the calculation of proportions.

Barbara's explanation, for all its incompleteness, was judged to be 'well-explained' in contrast to very partial explanations such as those of Danielle or Beth that were similar to those given by most other teachers. These explanations often related only to fractions as conceived of in the sense of cutting a single cake, or shading in regions of equal area in a shape. The only other teacher who gave a similarly rich, but also difficult to follow response was Alan, another connectionist teacher although from a different school.

This multi-faceted nature of the meanings and uses of concepts in numeracy are what makes the teaching of numeracy challenging. It can be appreciated how difficult it is to elicit the degree of complexity of teachers' meanings since many of these facets are known either implicitly, or not at all. The difficulty of accessing such knowledge is shown by Barbara's and Alan's responses; it seems possible that other teachers could, given time, construct similar networks of ideas.

Factors affecting gains
Overall figures for the 18 case-study teachers on the variables *fluency*, *scope*, *links*, *explanation* , *depth* and *understanding* were calculated and these variables were each in turn plotted against pupil gain scores.

In the case of *depth* i.e. the proportion of links that were explained in conceptual terms rather than by procedural (rule-based) connections, there was a moderate relationship with pupil gains, shown in Figure 4.3.

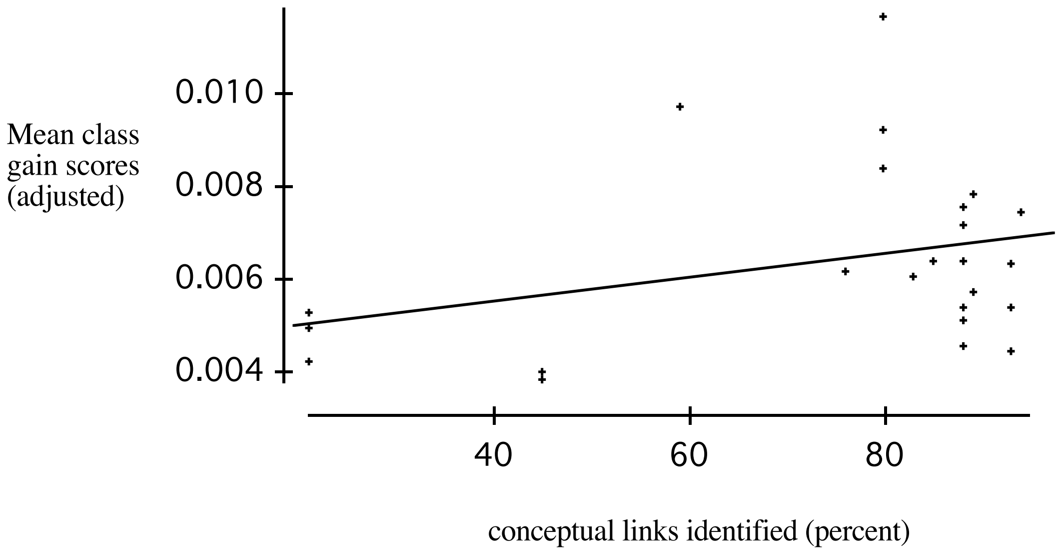


Figure 4.3: Percent of conceptual links identified by teachers plotted against adjusted mean class gain scores.

Figure 4.3 shows that the main reason for the relationship between depth and pupil gain is that teachers with a low proportion of conceptual links tended to have low gains. These teachers are predominantly those with transmission orientations, so this relationship will be discussed in section 4.5 which deals with the links between subject knowledge and teaching orientation.

However the graphs and regression analysis indicated that there was no clear relation between class gain scores and any of the other subject knowledge variables, *fluency*, *scope*, *links*, *explanation* , or *understanding*.

Within the area of *knowledge of content*, one might have expected that teachers who are highly effective in teaching numeracy might be more fluent in supplying numeracy concepts, and might be broader (or even perhaps more narrow) in their views about what numeracy includes.

One rather speculative explanation for this lack of relationship is that in calling into mind the constituents of numeracy, it sometimes seemed from reading the interview transcripts that teachers might be trying to remember chapter headings in their published schemes. This suggested that teachers' views of primary mathematics might be strongly framed by the structure adopted by these schemes.

Other teachers seemed to be trying to recall attainment target titles, as Algebra and Data Handling were sometimes mentioned. If teachers were trying to recall external structures rather than their own mental organisation of numeracy ideas, this would certainly explain any lack of relationship. It does however suggest that it would be interesting to explore significant influences on teachers' mental frameworks for thinking about numeracy.

Similarly within the area of *knowledge of relationships*, it might have been expected that the variable *understanding*, which related to the quality of the explanation given of the links between concepts would have had some relation with class gains, even if the number of *links* identified and the number of partial *explanations* provided did not affect gains.

Examining the data closely suggests that one reason for the lack of relationships between pupil gains and most of the subject knowledge variables may be that other interacting factors are of greater significance. For example the fact that some of the highly effective teachers of pupils in younger age groups did not provide very sound explanations suggests age group taught as a possible factor. Equally some teachers, with high mathematical qualifications themselves, not surprisingly performed well on knowledge of content and relations, whereas their students did not perform particularly well. For these reasons in sections 4.6 and 4.7 respectively, the effects of age group taught and teachers' mathematical qualifications will be examined.

4.5 Numeracy subject knowledge and teaching orientations

Given that *depth* was the one variable which, among the case study teachers, was related to gains in class mean scores, it is perhaps not surprising that this is also the only variable that seems to relate to teaching orientation, as shown in Table 4.1. The variable *depth* was defined as the percentage of explanations that were conceptual rather than just relating to a procedure without any apparent rationale.

Orientation	**Teacher**	**Depth (% of conceptual explanations)**	**Mean class gain**
Strongly Connectionist	Faith	94%	High
	Alan	93%	High
	Barbara	89%	High
	Anne	80%	High
	Carole	59%	High
	Claire	50%	(Year 1)
Strongly Transmission	Cath	45%	Low
	Elizabeth	21%	Low
Strongly Discovery	David	96%	Low
	Beth	76%	Low

Table 4.1: Teachers, orientation and depth of understanding of numeracy concepts

In spite of the small sample there does seem to be some suggestion that the teachers who best characterise a *transmission* orientation use a higher proportion of procedural (rule-based) links. This would be consistent with the beliefs associated with a transmission orientation about a view of numeracy as the acquisition of set routines and procedures, and supports findings in Chapter 3 that this tends to be associated with lower gains in class mean scores in numeracy.

The qualitative data illustrating the style of response in the interviews is even more salient than the percentages suggest. For example Elizabeth, in explaining the links between fractions and decimals focused on the procedure for doing the conversion. She then rather unusually linked fractions to time:

> *They will all reduce down to lowest terms, of sorts...I do a lot of converting from that to that ...so it's sort of conversion. I try not to link fractions and decimals...fractions are very much an area of their own...Fractions, that's times...well if they've got to add twenty minutes I find that children will say 2.2 hours ... instead of two and a third , and two and a quarter hours they will put it down as 2.4...their knowledge of time is abysmal and they cannot calculate time(Elizabeth Y5/6)*

Cath selected place-value and addition as her first pair of concepts to link, because:

> *Well, one of the things I've found with children - they do badly often if they don't have a good thing of place value, so that if you give them, say, a hundred and thirty nine plus sixty seven, they don't set it down right so you end up with ...total nonsense ...they just start at this end (the left) and it goes in order (Cath, Y5/6)*

She also connected multiplication and division to money:

> *...we do the four rules of money (Cath Y5/6).*

Finally fractions, decimals and percentages were connected together with an explanation that is partly procedural and partly conceptual:

> *Basically to me they're all the same - they're all a way of calculating, and the ratio as well...they're all a way really of collecting, separating and dividing things up ...It makes you think this doesn't it? It's the things you just do because it's too instant...you've done it for so long you forget why you are doing it, but you just do it - it's there (Cath Y5/6)*

Although the transmission teachers gave much more procedural answers, there was little difference between *discovery* and *connectionist* orientations in the percentages of conceptual responses. This may be due to the fact that discovery orientated teachers espouse the belief that pupils should develop conceptual links in mathematics and although their associated practices do not always foster this, the activities provided are usually intended to have this aim.

Nevertheless in the concept mapping interview the two *discovery* orientated and only moderately effective teachers in the case study sample gave rather brief and unelaborated, although not generally rule-based, responses:

> *understanding the value of a fraction, what is a quarter...1 out of 4...it can be linked with decimals... .5 is a half ... understanding that they are the same thing (David, Y5) .*

Anne and Alan, with *connectionist* orientations, roved around talking freely but not very precisely about the links between many different concepts. Alan used the words *conceptual* and *procedural* himself in some cases to describe his links. In the case of fractions and decimals he at least takes the idea of proportion as the common element:

> *That relationship between proportion and ratio is so fundamental to so many of the things that they do, the relationship between writing something as a fraction and it being the equivalent of what is written in per cent and*

> *also the equivalent to what is written as a decimal ... that ability to see that they all mean the same ..a proportion of something (Alan Y5/6).*

Thus the ways that teachers talk about the links between concepts in numeracy tends to reflect their orientation, except that the two *discovery* teachers were conceptual but unelaborated.

It was however clear that teachers in all orientations found it very hard to talk with clarity about the links between different concepts; where they appeared to give a quick answer, they seemed sometimes to be hiding behind received notions like 'parts of a whole' without considering whether these were helpful formulations.

4.6 Numeracy subject knowledge and key stage

It can be seen that the only two connectionist teachers with rather low scores on *depth* in Table 4.1 were Carole and Claire, both Key Stage 1 teachers. This suggests that year group taught might be a significant feature, perhaps because many of the areas of mathematics involved in the interviews concerned with subject content knowledge might be beyond what some teachers would normally be teaching.

When the results of the concept mapping interview were plotted against the year groups of pupils taught, there was not a strong relationship overall between the mathematical subject knowledge of the teachers and the year groups of the pupils they taught. In particular some of the best results on the concept mapping were gained by Year 3 teachers such as Danielle and Beth who demonstrated comparatively sound subject knowledge (but only moderate pupil gains).

However when the group of Key Stage 1 teachers (Year 1 and Year 2) were grouped together and compared with the Key Stage 2 teachers, as shown in Table 4.2, a slight indication of differences in some of the aspects emerge. As can be seen, any differences lie in the *explanation* and understanding variables, which refer respectively to the proportion of links explained and the quality of the explanations offered, rather than in the variables of *scope*, *fluency* or *depth* (i.e. tendency to explain links in terms of concepts rather than procedures). However the size of the Key Stage 1 sample was very small (only 6 teachers), with only 13 in the Key Stage 2 sample (one of the 18 teachers taught classes in both key stages), so that this result is very tentative.

	Mean for KS 1 (6 teachers)	Mean for KS2 (13 teachers)
fluency	15	17
scope	7	7
links	15	16
explanation	42%	57%
depth	70%	79%
understanding	16%	31%

Table 4.2: Subject knowledge variables for KS1 and KS2 teachers

However the effect can be illustrated by quotations from the interviews of the Year 2 teachers in the case study sample in the focus schools.

As previously noted two Year 2 teachers, Carole and Erica, found the questions relating to decimals and fractions to be somewhat traumatic. Carole, a *connectionist* teacher :

> *I hate those two (decimals and fractions)...twelve sixteenths times three eighths or something ghastly.(Interviewer: If I were to ask you to convert a seventh to a decimal, would you feel comfortable?) Shock, horror...I obviously had this ghastly teacher at school who gave me this thing about fractions and decimals ... but you see percentages, I've come to terms with those more because we use them a lot at school...there's something that I could do a lot better ... I need to just have time to sort it out. (Carole Y2)*
>
> *I feel safer with fractions than decimals which I know is mad really....I've got into a pickle over that (converting 1/7 into a decimal)...I never know where to put the dot on all the points, that's why. I wouldn't teach it ... I've been totally thrown...I hate decimals (Erica Y2)*

It is therefore clear that some teachers of younger children have real problems over subject knowledge, but it is not clear how much this affects their effectiveness. Carole was highly effective in terms of the gains for her Year 2 pupils; indeed she had almost the highest gain for that age group. However, there might be an impact on effectiveness if such teacher moved to teach older classes.

It is perhaps worth noting that Erica, who had no strong orientation, has an A-level in mathematics, although taken some years ago, and Carole specialised in mathematics in her 1-year PGCE. This takes us onto the question of the relation between subject knowledge and formal mathematical qualifications.

4.7 Teachers' mathematical qualifications

Case study teachers' mathematical qualifications
The association of *depth* of understanding of numeracy concepts with mean class gain scores leads to the question of what sort of mathematical background and qualifications might have led to this conceptual understanding. As a first attempt to examine this it is helpful to see the qualifications of those case-study teachers who represent strong orientations, as set out in table 4.3

Orientation	Teacher	A level Maths?	Relevant degree?
Strongly Connectionist	Anne	No	No
	Alan	No	Biology
	Barbara	No (studied but not examined)	No
	Claire	Yes, grade C	Mathematics
	Faith	Yes, grade D	No
	Carole	No	No
Strongly Transmission	Cath	Yes, distinction	Science
	Elizabeth	No	No
Strongly Discovery	Beth	Yes, grade unknown	Mathematics
	David	No	No

Table 4.3: Teachers, orientations and mathematical qualifications

Thus the formal mathematical qualifications of these particular case-study teachers seem to show little association with their teaching orientation.

Full sample (90 teachers) mathematical qualifications
In order to provide further evidence of the relationship between teachers' mathematical qualifications and pupil gains on the numeracy test we turned to the full sample of 90 teachers. As previously indicated, data on the teachers' mathematical qualifications was gathered from questionnaire data.

It was assumed that all teachers had achieved at least the equivalent of O-level/GCSE mathematics at grade C, since this has been a requirement for teacher training for a substantial period. Eighty-eight out of the 90 teachers supplied information on their more advanced qualifications:

- 3 had studied for a mathematics degree
- 13 had studied 'A' level mathematics(including the three with mathematics degrees)

- 11 of the 13 had passed 'A' level mathematics (one of these having two 'A' levels in mathematics, pure and applied).

Out of the 11 who had passed 'A' level mathematics, 8 were prepared to reveal their grade level:

- 7 out of these 8 teachers had achieved at least a grade C.

The box and whisker diagrams below show the relative class gains (adjusted - see Appendix 1.3 for further details) for two groups of teachers: in Figure 4.4, the 75 teachers without an 'A' level in mathematics (N) compared with the 13 teachers who *studied* 'A' level mathematics (Y); in Figure 4.5, the 77 teachers without an 'A' level in mathematics (N) compared with the 11 teachers who studied *and passed* 'A' level mathematics (Y). (The 'box' part of the diagram shows the range of gains within which the middle 50% of the classes lay. The bar across the box indicates the median class gain score for that group of teachers. The 'whisker' sections indicate the full range of gains, with the range for the top 25% of class means in the whisker above the box and the range for the lowest 25% below it. Any 'outlier' points lying clearly outside the main range are indicated as small circles).

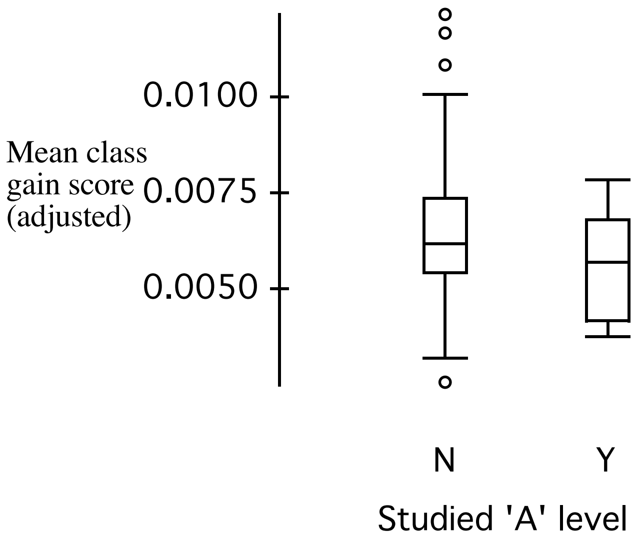


Figure 4.4: Mean class adjusted gain scores for teachers studying or not studying 'A' level mathematics

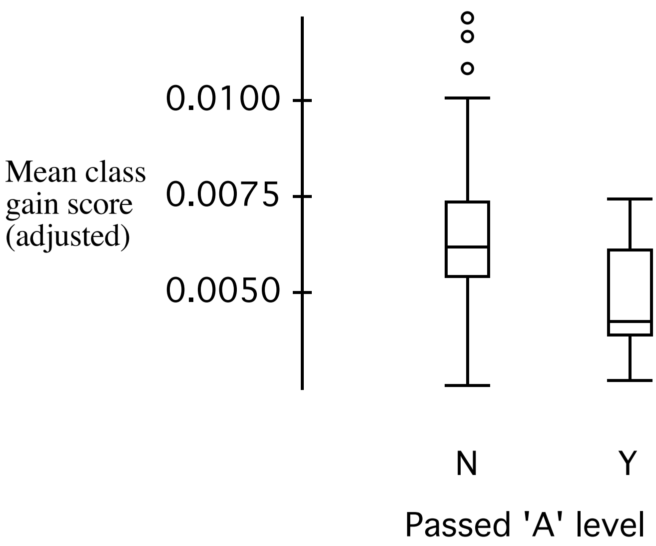


Figure 4.5: Mean class adjusted gain scores for teachers passing or not passing 'A' level mathematics

As can been seen in both cases the median class gain was lower for the teachers who either studied and/or passed 'A' level mathematics.

If the data relating to possession of a degree in mathematics is examined, the sample becomes smaller still with only 3 teachers with a mathematics degree. One of these, Claire, was teaching Year 1 and thus no pupil gain scores are available, since Year 1 pupils were judged too immature to be tested in October and were tested only in April. Although the scores of her class in April were high, it is difficult to judge the significance of this without initial data.

The other case study teacher with a degree in mathematics, Beth, had comparatively low gain scores and was classified as moderately effective, although towards the top of this group.

Thus, while firm generalisations cannot be drawn from such a small sample, it is certainly clear that it cannot be assumed that either a qualification in 'A' level mathematics or a mathematics degree will necessarily be positively associated with larger pupil gains.

Discussion

Why does this finding not support what common sense might suggest, i.e. that higher mathematical qualifications should be associated with higher pupil gains? One possibility suggested by teachers' comments was that the nature of what the teachers felt that they had learnt at higher levels of mathematics was perceived as unconnected with the content that they were teaching:

> *Degree maths is not relevant to primary maths at all. ... I absolutely hated my maths degree, it was such a big difference from school. ... (Mathematics)*

is a subject I coasted through at school, I never had any problems with it and suddenly there was this big mountain. (Beth Y3)

This observation made by Beth (who had a *discovery* orientation) was supported by the fact that although her degree was only 5 years previously, she found it difficult in the interview to explain the links between fractions and decimals, repeated subtraction and division, and confessed to problems over estimating the size of metric units.

Similar comments were made by teachers about A-level mathematics :

I got an A in A-level maths so I do not see any difficulty but if anyone asked me what I taught best I would say English; I do not think mathematics is my strength (Dorothy Y4)

Dorothy, like Erica in the previous section, seemed to have been able to achieve success at A-level without having developed a sound and connected understanding of basic mathematical concepts. (Neither teacher had a strong teaching orientation, or high subject gains.) The question as to why teachers perceived A-level mathematics as unconnected with their understanding of numeracy is explored further in section 5.5 of Chapter 5.

4.8 Correctness, adequacy and excellence of subject knowledge

The subject knowledge of mathematics and numeracy among primary teachers has been felt to be a concern (e.g. Ofsted, 1994). It is important to analyse what aspect of subject knowledge is thought to be weak.

At one level it may be that the concern is that the weakness of teachers' mathematical knowledge and skills affects the correctness, or at least the scope of what is being taught in the classroom. In none of the 84 lessons were there any significant mathematical errors made by teachers, and in only two were there occasions when teachers found themselves to be clearly limited by their knowledge:

A Y6 teacher was introducing the idea of the median as a way of determining the 'centre' of a distribution of numerical data . A list of values was put on the board and the teacher worked through how to find the median. (With a small sample of numerical values this is by arranging the numbers in order of size and selecting the middle one.) Occasionally a list containing an even number of values was put up, but once the teacher started to work out the median a further value was added to make the number of values odd. When asked why this was done, the teacher admitted to not knowing how to calculate the median of an even number of values, *because there is no middle one. (It is usual to take the mean of the two middle values.)*

A Y2 teacher was working with calculators and the children got very interested in the eight digit numbers they could show on the calculator displays. The teacher was reluctant to respond to the children's requests to know what the numbers were and later confessed that she was not confident in reading these large numbers.

Although these examples do demonstrate important gaps in subject knowledge, neither would seem to be especially damaging or difficult to retrieve. Most teachers admitted that there were sometimes questions to which they did not know the answer, but that they had people whom they could ask, or books they could look up. Some confident teachers were unashamed about losing face, and made a positive learning opportunity of it, encouraging pupils to find out the answer before they did.

Similarly in the interviews, no mistakes were made but, as noted earlier, two of the 18 teachers (both Y2 teachers) confessed that they could not immediately remember how to convert 1/7 to a decimal. However despite the panic it caused them, they both felt confident that they could find out what the method was.

All case study teachers were asked about any problems they felt they had with teaching numeracy that were due to gaps in their subject knowledge. All teachers felt that they were able to cope quite adequately. That is perhaps not surprising given the fact that the case study teachers were selected as the most effective in a group of effective schools.

Nevertheless it was clear during the interviews that many teachers found it very difficult to talk about the links between mathematical concepts that they were teaching in their classrooms. One particularly difficult example was that of fractions and decimals, where even the teachers who were best able to discuss the multi-faceted nature and applications of these ideas were still far short of any degree of clarity.

Classroom observation also suggested that some teachers were giving pupils a very limited and fragmented view of mathematics. With the exception of the *connectionist* teachers, neither teachers nor pupils seemed to be able to link together satisfactorily the various parts of the curriculum.

Thus one could judge most of the teaching adequate, in the sense that pupils were learning specific skills. However if one is looking for excellence, in that the teaching should provide pupils with enthusiasm about the subject, knowledge about its nature and how it can be applied, and most importantly a flexible use of ideas and skills in tackling challenging problems, either in context or of a mathematical nature, then much of the teaching falls short, even for some teachers in the highly effective category.

4.9 Discussion and implications

Lack of evidence of any positive association between formal mathematical qualifications and pupil gains should not be interpreted as suggesting that mathematical subject knowledge is not important. What would appear to matter, as the later section on mathematical subject knowledge indicates, is not the level of formal qualification but the *nature of the knowledge about the subject* that teachers have.

Ball (1991) argues that correctness, meaning and connectedness are requirements of teachers' mathematical subject content knowledge for teaching mathematics for understanding. Although not corresponding exactly to our categories there would appear to be some similarities. The evidence from our research suggests that there was little to distinguish between the teachers in terms of their understanding of the content of the numeracy curriculum as far as correctness and a very straightforward sense of meaning were concerned. This is not to say that for individual teachers there were not pockets of number content knowledge that they were unsure about.

However, the connectedness of their mathematical knowledge in terms of the depth and multi-faceted nature of their meanings does appear to be a factor associated with pupil gains.

One implication of this is that teachers do not necessarily need 'additional' mathematical knowledge. 'More' is not necessarily 'better' in terms of helping pupils understand mathematics. Rather, primary schools teachers may need to develop a fuller, deeper understanding of the number system in order to effectively teach numeracy.
Aspects on which this understanding might concentrate we suggest could include:

- The structure and evolution of the number system and the conceptual links between different aspects of number. For example: how place value, zero, powers of 10 and the relationship between these provides a conceptual framework for decimals.
- The multifaceted nature of meanings and applications of mathematics operations and notation. For example, that the mathematical operation of subtraction can be used to calculate a variety of situations including finding the difference, taking away, counting on.
- The relationship between identical surface (i.e. notational) features and different underlying concepts. For example, that the fraction notation

system can be used to denote parts of unity, parts of a collection, ratio, proportion, scalings, a point on the number line, etc.

- The relationship between different surface features and similar underlying concepts. For example, that the notation of fractions, decimals, percentages or ratios could all be used to represent the same situation, the appropriate symbol system being selected according to purpose. However, traditionally, particular representations are used for particular purposes.
- Understanding of and facility in moving between different representations. For example, being able to convert between fractions, decimals and percentages where appropriate and to explain the reasons for equivalence.
- Appreciating the limitations of notational systems. For example, understanding the difference between rational and irrational numbers and why the decimal notation system does not allow all rational numbers to be expressed as a precise (terminating) decimal.
- The application of numbers to shape and space, data handling and measurement. For example the use of percentages to compare proportions in samples of different sizes, the way irrational numbers arise in geometry, the dependence of all measurement on the concept of ratio.

4.10 Knowledge of pupils

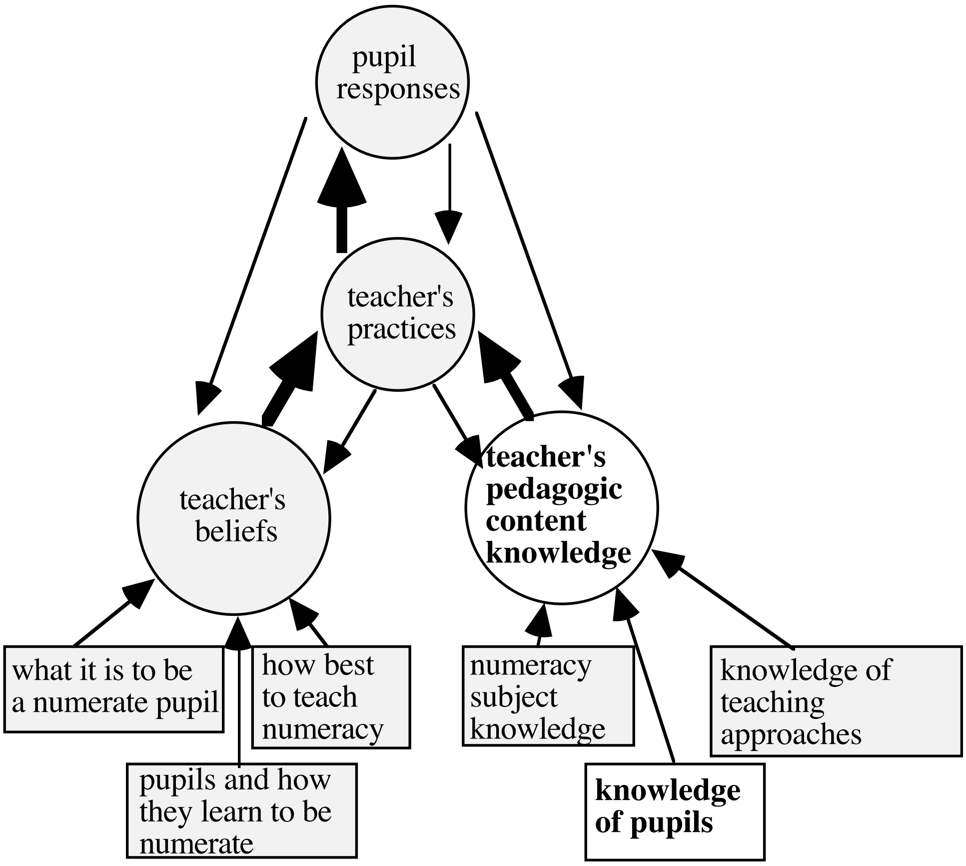


Figure 4.6 Focus on the part of the model concerned with a teacher's knowledge of pupils

Alongside mathematical subject knowledge and understanding we also considered it important to examine how the teachers made sense of pupils' learning, both in general and for individuals. So the second strand of pedagogic subject knowledge explored was teachers' knowledge of pupils' understandings, attitudes and approaches to learning mathematics and becoming numerate.

In order to examine these areas, a technique of eliciting *personal constructs* was used. This technique involved an extended interview with each of the 18 case study teachers, working with pupil names randomly selected from the class taught. Presenting these names in groups of three, the teacher was asked to identify some way in which two of the pupils display behaviours in their understanding of numeracy that the teachers considered to be significantly different from the third pupil. The process was repeated both with the same set of three names and with different sets of names until no new *constructs* were forthcoming.

A set of pairs of opposites was thus elicited from each teacher. For example 'confident' v 'insecure when working alone'. The teacher was then asked to order each pair of constructs, indicating which of the pair they thought more significant in helping pupils become numerate. Finally the full set of pairs of opposites was ranked in order of importance.

This data was again analysed both quantitatively and qualitatively. Using computer packages the sets of constructs were grouped and examined for patterns of similarity and differences among the teachers. These could then be compared with pupil gain scores to see if there were associations between any particular types of constructs and pupil attainment.

Transcripts of the interviews allowed further analysis of the data to explore qualitative differences in the way the teachers considered pupils and the relationship with the orientations towards teaching mathematics that the teachers displayed.

4.11 Teachers' knowledge of pupils: aspects identified

From the analysis of the constructs, eight main areas of attention emerged. These eight areas are described below and presented in descending order of popularity. That is, statements about pupils' attitudes were elicited most often from the teachers and statements about pupils' interpersonal skills least often.

- *pupil attitude*: in particular whether pupils displayed confidence in mathematics and the extent to which they were prepared to take risks in lessons and explore ideas.
- *pupil ability in general:* constructs here included how quick pupils were to learn new ideas and whether or not they were a generally 'able' pupil.
- *pupil knowledge and understanding of numeracy*: areas identified and talked about included knowledge of tables and number bonds, the ability to carry out standard algorithms, whether or not pupils could apply their mathematics.
- *pupil ability in mathematics*: in contrast to *pupil knowledge and understanding of numeracy* statements made in this category focused on pupils mathematical aptitude in general terms, for example describing a pupil as being mathematically able or not good at mathematics.
- *pupil approaches to mathematics*: this covered such aspects as whether or not a pupil was systematic in their approach to mathematics or whether they were careless.
- *pupil needs*: these were constructs where the focus was on what the teacher had to provide for the pupils, for example identifying one pupil as needing a lot of extension work or another as requiring more practical work.

- *pupil background and experiences*: this small group of constructs focused on external circumstances that might have influenced a pupil's numeracy, for example whether or not the pupil was supported at home or whether or not they were 'street-wise'.
- *pupils' interpersonal skills*: this very small group of constructs paid attention to whether or not pupils were prepared to come and ask for help and how easy they were to work with on a one–to–one basis.

4.12 Teachers' knowledge of pupils and pupil gains

For each teacher the percentage of their *constructs* that came into each of the above eight categories was calculated. These were then checked against the mean gain scores for the classes the teachers taught to see which, if any, of the groups of *constructs* was associated with pupil gains.

Only two of the eight areas revealed an association with pupil gain scores. Both

- *pupil attitude* and
- *pupil approaches*

showed a moderate positive association with pupil gain scores. The greater the percentage of statements teachers had made about either of these aspects the higher the mean class gain score.

At first this seemed like a surprising result. Existing research would suggest that teachers with good knowledge about individual pupils' attainment in mathematics achieve higher attainment. So why was the extent to which teachers identified pupils' knowledge and understanding of numeracy *not* associated with higher pupil gains? In order to shed some light on this question it is helpful to examine the case study teachers' assessment and record keeping practices.

4.13 Orientation, assessment and record keeping.

The *connectionist* oriented teachers, who by and large were the group of case study teachers with high mean class gains, were marked by the attention they gave to carefully monitoring pupils' progress and keeping detailed records. They built up at both the class level and, when appropriate, in terms of individual pupils, detailed profiles of current levels of attainment in mathematics.

Barbara made clear how she used assessment to monitor both the teaching and learning and to inform ongoing planning.

> *So it's all sort of assessment and focus teaching all the time. ... I assess every day - what activities have gone on and where each group goes next. ...*

The assessment tests–we give them at the beginning of the topic, spend 3 weeks working through all the concepts and so on and the skills, and give them the same test at the end of the topic. And I've looked to see what marks the children got at the end and it was just two children that hadn't made any improvement.
I keep mental maths records. I keep notes of the strategies we discussed in that session and of course you can't assess it because we discussed it so it would have to be another session when you gave them similar problems and you can assess if a child had used this strategy at all (Barbara Y6)

Ongoing assessment and detailed record keeping allowed Anne and Alan to provide learning experiences to meet the specific learning needs of individual and groups of children within their classes.

Every piece of work I do, I just keep a sheet like that ... and I write my own notes on there if a child has a specific problem. ... My planning, my search to find the most suitable method of teaching a child or children and that just comes about from my own experience and my observations and my constant assessment that I use. (Anne Y234)

I'm ... trying to get at least something from everybody, even if it's only one little sentence, in a lesson. I'm checking on the work that they give me. (Alan Y56)

Similar patterns of a mix of formal monitoring of progress, assessments specifically designed to assess pupils' knowledge of particular curriculum areas and ongoing teacher assessment were indicated by other connectionist teachers.

I think it's a kind of, continuously gathering and processing information about children's understanding. It sounds weird but it's kind of organic in a way. Because you are constantly getting feedback about how they're going. ... So it is a matter of constantly gathering through discussion, questions, helping them with their work, marking their work, how their progress is going. (Claire Y1)

I tend to make notes if I find a child has difficulty in a certain area I'll make a note of it and work out a programme so that they can have more experience of that. (Carole Y2)

(Interviewer: What kinds of assessment do you use?)
Teacher assessment, self assessment, monthly tables tests, mental arithmetic books that we do every other week, mental tasks so all mental problems that we do every alternate week in mental arithmetic between maths lessons over and above the mental maths lesson we have on Friday. Children are encouraged to assess themselves, problem solving. (Faith Y4)

At school A, one of the schools with consistently high pupil gains, careful assessment and monitoring had been implemented throughout the school. At the school level standardised tests were used to set pupils for mathematics, according to ability. Within year groups teachers regularly discussed the progress individual children were making.

> *We have tests throughout the school. ... It is a (commercial) test that we are going to use, actually we use the (National Curriculum test) results for Year 3. ... We try to pass things round and then we have a lot of discussion about the problems that children have, how we can solve them. (Anne Y234)*

> *I use continuous assessment from talking to the children, and from listening to what is going on and looking at the work they are doing. We are moving over towards using more objective testing, more regularly ... so that we have got some sort of standardised test as well as our procedural work that is going on. It is a mixture really of continuous assessment, getting the children to feed back. I mean a lot of it is ephemeral but punctuated at significant times in the year by set chances for the children to show what they have taken on board. (Alan Y56)*

In contrast, the *transmission* orientated teachers appeared to have a different attitude to assessing. They were more concerned to check up that what had been taught had actually been learned, rather than acknowledge that what pupils could do or understand might not match what was taught. Assessment was a retrospective process, rather than one that fed into future plans.

> *They have what we call a weekly revision paper which they take home every week which covers all the work we have done so far...it's repetitive practice, so once the parents have stopped you know joining in, that gives me a very very good indication of if they are keeping up and I can quite quickly pinpoint the areas where I need to go over again. ... I was teaching long division long before children were ready for it ... I can sense now when they're ready and I can also sense if I've gone too fast ... I do teach a lot by instinct. (Elizabeth Y5/6)*

Cath used class revision sheets built into the mathematics text books to test the pupils' grasp of the topics taught. Children who had not understood a particular topic were given further practice in this area.

> *When we've done all the topics with the children then they use the check up sheets and if ... they don't get it then we do some reinforcement work ... which is virtually the same but they use different numbers, then they have a go again (Cath Y4/5)*

This metaphor of 'checking up' on the children to see if the teaching had worked (and providing further instruction if not) was consistent with a *transmission* orientation and a focus on teaching over learning.

4.14 Discussion and implications

It would seem that rather than not paying attention to pupils' knowledge of numeracy, the strongly *connectionist* teachers were paying a great deal of attention to it. However, rather than trying to hold this information in their heads they were documenting it carefully and then using it to inform their teaching.

Freed from trying to hold a great deal of information about pupils in their heads, it would seem that these teachers were then able to concentrate on the aspects of learning mathematics to do with pupils' attitudes and approaches, both of which are rather harder systematically to assess and record.

The strongly *transmission* orientated teachers used assessment for a different purpose, to check up on teaching rather than to inform teaching. This is a subtle difference. Assessment for checking up focuses in the main on assessing pupils after a sequence of teaching. Any gaps in understanding revealed through this are likely to be dealt with by revisiting what had been taught, in a similar fashion.

Assessment to inform teaching has more of an on-going dimension to it. Assessment may occur at the beginning of a topic to help in planning and the use of assessment within a teaching sequence feeds back into the teaching as it progresses.

It would seem that teachers should be encouraged to review their assessment practices at both the class and school level and examine the extent to which it is genuinely used to inform teaching.

4.15 References for Chapter 4

Ball, L (1991). Approaches to the professional development of teachers. In L. Bell & C. Day (Eds.), *Managing the Professional Development of Teachers* Milton Keynes: Open University Press.

Chapter 5: Professional development experience and effectiveness

5.1 Introduction

In Chapter 3 we described how the beliefs and practices of highly effective teachers seem to differ from those of other teachers, and in Chapter 4 we explored the relationship between pedagogic subject knowledge and effectiveness. As demonstrated there, it is hard to identify links between highly effective teachers and specific items of subject content knowledge. There is, however, evidence that highly effective, *connectionist* orientated teachers tended to be keener to elaborate on the links between different numeracy concepts than did other teachers.

In this chapter we examine what it is in terms of training and experience that seems to contribute to forming the beliefs and practices that accompany effectiveness.

We collected data from questionnaires and interviews on factors suggested by previous research that might be influential. We also examined factors arising out of the data analysis in that we coded all our interviews with teachers to trace anything that they reported as having had a significant influence on their thinking. It should be kept in mind that such reports provide evidence of what teachers perceived as being influential, while the actual cause of change may lie elsewhere.

The factors thus identified are discussed under these sub-headings:

- 5.2 Continuing professional development and pupil gains
- 5.3 Continuing professional development and teaching orientation
- 5.4 Initial training
- 5.5 Own experience of mathematics as a pupil /student
- 5.6 Other teacher factors
- 5.7 School influences
- 5.8 Discussion and implications

The specific data sources drawn on are described in each section.

5.2 Continuing professional development and pupil gains

In section 4.7, as part of our investigation of teachers' subject content knowledge, we examined the available data on teachers' formal mathematical qualifications and its association with pupil gains on the numeracy test. Data on the teachers' continuing professional development (CPD) was also gathered to further illuminate the relationship between pedagogic content knowledge and pupil gains.

Data on CPD was available from two sources:

- questionnaire data from the full sample of 90 teachers, 88 of whom provided background information on the number of days of CPD in mathematics over the previous year, the range of activities engaged in, and any extended programmes (20 day, certificate, diploma or masters) studied, both in mathematics and in other subjects;
- interviews with 33 case study teachers, 18 from the focus schools and 15 from validation schools, which enabled them to talk about aspects of CPD that had been significant for them.

From the questionnaire data on 88 teachers, it emerged that:

- 26 percent (23 teachers) had engaged in some form of extended CPD, comprising:
  - 8 percent (7 teachers) who had engaged in extended CPD in mathematics education;
  - 18 percent (16 teachers) who had engaged in extended CPD in subjects other than mathematics education.

The box and whisker diagrams in Figures 5.1 and 5.2 show the relative class gains for two groups of teachers. (The gain scores have again been adjusted to account for the fact that classes with high initial assessment scores would find it harder to make large gains - see Appendix 1.3.)

In Figure 5.1, the mean class gain scores of teachers with experience of extended CPD in mathematics education (1) are set alongside the class scores of teachers with no experience of extended CPD in mathematics education (0). In Figure 5.2, the mean class gain scores of teachers with experience of extended CPD in subjects other than mathematics education (1) are compared to those of teachers with no experience of extended CPD in subjects other than mathematics education (0).

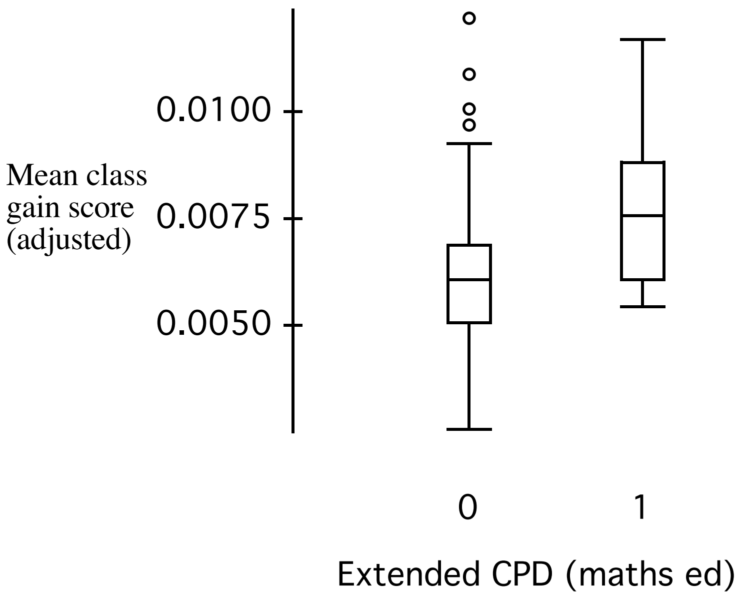


Figure 5.1 Relation between CPD in mathematics education
and adjusted class numeracy gains

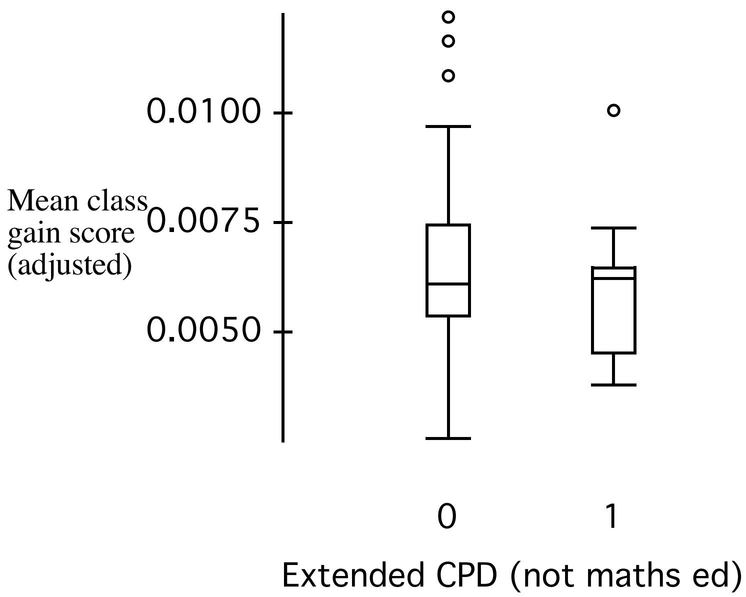


Figure 5.2 Relation between CPD in subjects other than mathematics education
and adjusted class numeracy gains

As can been seen from Figure 5.1, the median class gain was higher for the classes with teachers who had engaged in some form of extended CPD in mathematics. In fact, 75 per cent of the mean scores for these classes were above the median score of the means for the remaining classes. Notice also that the tail of lower scores is much shorter for those classes whose teachers had engaged in extended CPD.

In contrast to this, Figure 5.2 shows that the median score for those teachers who had engaged in extended CPD in subjects other than mathematics education was only marginally above that for the other teachers.

This suggests that experience of any kind of extended CPD is not in itself sufficient to ensure that teaching of mathematics is improved. While there might be transferable skills that cut across subject areas in terms of improving pupil attainment it would appear that teachers do need to be supported by mathematics-focused CPD in putting these into the context of mathematics teaching.

What these results cannot indicate is whether the better pupil gain scores for the group of teachers who had been on extended CPD in mathematics education was a direct result of the training they had received, or if it was the more effective mathematics teachers who were attracted to engage in extended CPD in mathematics. However the data from the case study teachers does suggest that certain aspects of extended CPD had affected their practice, as is shown in the next section.

5.3 Continuing professional development and teaching orientation

Of the five highly effective strongly connectionist orientated teachers in the focus schools, Anne, Barbara and Faith had all engaged in extended CPD in mathematics education.

None of the strongly transmission or discovery orientated teachers, who were only moderately effective, had been involved in extended CPD in mathematics education.

Many teachers of different orientations spoke about one-day workshops or INSET programmes they had attended, but while they were often said to be 'useful', no teacher indicated any way in which they had had a significant influence. Analysis of the data for all 90 teachers of class gains against the number of days of CPD in mathematics education engaged in during the previous year demonstrated that only those teachers who had engaged in extended CPD in mathematics education (at least 15 days) were highly effective. The class scores of teachers having had one, two or three days of CPD in mathematics in the previous year were essentially indistinguishable from those teachers reporting no days.

Four of the five highly effective *connectionist* orientated teachers, Alan, Anne, Barbara and Faith, all identified an emphasis on the importance of working with pupils' meanings and understandings as significant elements of the CPD that they had engaged in. Moreover, Anne aside, they indicated that they had not been aware of the importance of developing pupils' meanings and mental strategies until the CPD made them focus upon this. This awareness had been raised both by examining the teachers' own understandings and strategies, as well as those of pupils'.

Barbara and Faith, who had been on 20-day GEST programmes, were very enthusiastic about their experiences. Faith reported that it '*encouraged me to think* *more about maths and how you can bring those ideas to the rest of the staff*'. She welcomed the fact that about a quarter of the programme had been pitched at the development of their own learning:

> *There was a lot of 'hands on' activities, problem-solving, but not as my PGCE had been, geared to activities for 8 and 9 year-olds. It was actually geared to our own understanding...which was challenging, which put you as a learner rather than a teacher, which was great, fantastic and fun (Faith Y4).*

Both mentioned how their awareness had been raised through professional development activities that had either focused on the teachers' own mental strategies or required them to work with pupils in this way:

> *One of the tasks that we had to do there was to take children away and to talk about it (methods of calculating mentally) and they first of all did it on us, which was absolutely fascinating because we were all teachers of maths, most of us subject co-ordinators, and we were given mental problems that we had to work out ... half of us there were absolutely gob smacked with these ways that people were doing it which made absolutely no sense to us at all, ... I couldn't understand any of that going on and it was the fact that we'd all had different maths experiences and yet eventually we did come to the same problem, but it was the talking about how people worked it out (that struck me)... I went across the school and I chatted to able, middle, and below average children in each year group and just set them some mental things and just talked about it, but it was the talking about it that showed me the ways that they were doing the mental arithmetic and that's something that I decided to focus on as a maths co-ordinator and so have given, well encouraged, all the other staff to do that. (Faith Y4)*

> *So that being aware of the properties of number which you often find out through working on number patterns enables the imagery to be built up and therefore enables you to work much more quickly...(Interviewer ... This strong sense of imagery, it that something you have always been aware of having yourself?) No I haven't, I haven't been aware of it at all. When I went on the 20 days maths programme some years ago we had a tutor on one of the sessions who asked us to do a mental calculation in our head and she said, who did it this way, who did it that way and she explained, and I realised I'd actually done an operation in my head..... I'd actually seen the sum whatever it was, long multiplication or whatever and I'd actually worked through the processes in my head. I suddenly became aware of the importance of imagery and how much quicker you can be at working things out... how much quicker and fun it is to manipulate the numbers in your head and move them about...and I realised then...I've really worked on it since if you like. ... I think that teachers need to be aware of the importance of it .(Barbara Y6)*

This suggests that both drawing teachers attention to the variety of mental strategies and asking them to listen to pupils' methods of mental calculation can be very effective methods of raising their awareness. This was supported by Danielle, an effective teacher who was not easily characterised as having one of the three orientations. She spoke about the impact on her practice of similar methods used in an Open University course *Developing Mathematical Thinking*. Like Faith, she also commented that she now listened more to pupils and built on their responses.

Barbara also drew attention to the broader issue of the importance of mental imagery in the process of becoming numerate, and how becoming aware of this had widened her approach from one that concentrated mainly on correct performance of procedures:

> *I wasn't taught with practical maths, I was taught formally. So I've never built up any imagery playing about with numbers, or playing with multilink or multi-base and so on. (Interviewer - So do you think that awareness raising that came through the 20 day programme has actually filtered through?) Oh absolutely, yes. It's changed all my attitude to the way maths is taught... Everything I do is a way of getting a picture in your mind...All my teaching altered since that was brought home to me...I would have taught them a set way of doing your long division and the algorithms that I learned at school, not bothering about what it meant, ...but (in a recent topic) they all did it in a way they could understand. So they all had different ways of doing it, but it made sense, and if you gave them one now, they'd have a good go at it because they would remember the picture...Whereas if I'd taught it before the programme it would have been just a sum...(Barbara Y6)*

This would seem to be a convincing testimony from a teacher whose Y6 class had very much higher gains on the Y5/Y6 test than any other class. Together with the data from other teachers who had engaged in extended CPD it suggests that the opportunity provided by an extended professional development programme in mathematics education to discuss and examine the basis of mathematical learning and understanding can have a powerful effect in raising standards.

Alan saw the development of his views about teaching as '*a gradual evolutionary process driven by necessity*'. He had not undertaken any extended CPD in mathematics. However he did refer to the positive effect of CPD he had attended at King's College London about five years previously on research into primary children's thinking in science (the SPACE project). He said this had:

> *...made me think...the notion that you must really know where the children are before you carry on...building on children's preconceived ideas rather than assuming that the child is a blank piece of paper (Alan Y5/6).*

When asked what other training he would like on mathematics he re-inforced this appreciation of research knowledge:

...more the opportunity rather than training, the opportunity to go along and hear some up-to-date research expanded...an easy way into research for practitioners in the classroom.

In regard to subject knowledge it was a global understanding of the significant ideas that he wanted:

...important shining lights in mathematics that every mathematician ought to know (Alan Y5/6).

While he recognised that he had an incomplete grasp of some subject knowledge detail he was confident that he knew the people to ask or the books to read and could build up his own confidence without the need for any programme.

The extended programme that Anne had attended several years previously was not a 20-day programme but 'a maths professional studies advanced programme' that had taken place over two years but only in the evenings. Anne recounted what she could recall of the programme:

They would maybe take a topic, say data handling or fractions or something...and set you a task and you would go away and prepare your own investigation and your own observations, and relate it back to the teachers running the programme...And then we would just discuss, and we had things like videos and tape recordings of children working. (Anne Y2/3/4)

Anne did not attribute her connectionist beliefs and practices to the effect of this programme, but said "*To tell you in all honesty it's the way I've worked ever since I can remember*". It is always difficult for people to remember if, how, when and why their ideas have changed, so it is not clear whether the programme had any influence on her, and if not whether this was due to the quality of teaching, the type of activities or the timing of the programme in twilight hours. Anne had also completed two certificate programmes in science teaching, and had done 'lots of inservice training' in her own time while working part-time with the county support team.

5.4 Initial training

Although it did not form a particular focus of the data-gathering, all 33 teachers who were interviewed were asked about the impact of initial training on their teaching of numeracy. Analysis of their responses revealed little association between initial training and teachers' effectiveness or orientation. Teachers rarely indicated that they *perceived* their initial training as an important feature in their development as teachers of numeracy, perhaps because many of the teachers we interviewed had considerable experience and therefore '*that sort of thing was in the dim and distant past*'.

Two of the five highly effective connectionist teachers volunteered that their 1-year PGCE primary programmes were so rushed that there was insufficient time available to properly prepare for mathematics teaching:

> *Fractions and decimals cause me so much panic ...I just need the time to sort it out..(Interviewer: What about your teacher training? Were any of these areas tackled then?) No, I did a PGCE (with maths as main subject) ...The two maths teachers were brilliant, ten times better than anything else ...(but) there just wasn't time to go into things like fractions and decimals...I don't see how we could have, there was such a pressure on the programme anyway. ..A year - ridiculous really isn't it?(Carole Y2)*

> *I only did a PGCE so ...I had minimal subject (input)....but then that was par for the programme ...for any of the subjects ... (Faith Y4)*

Faith later when pressed admitted that she did recall one session on her PGCE when students discussed their different mental strategies for working out a subtraction, but her perception of the main input she had derived from her PGCE mathematics training was a sense of:

> *the wealth of resources that are now available...that can enrich their mathematical learning (Faith Y4).*

Two of the other highly effective connectionist teachers, Anne and Barbara, had Teachers' Certificates from 3-year primary teacher training programmes, followed by additional certificates or diplomas, and in Anne's case a later science BSc degree. Neither made reference to their initial training, which was in each case over 20 years ago. Only one remark about lack of time for mathematics was made regarding 3 or 4 year primary training programmes such as the BEd, and that was from a more recently trained teacher.

Interestingly of the six highly effective teachers (five connectionist and one other), neither Alan nor Alice had had any initial training about teaching mathematics as both were secondary trained, through a science PGCE and BEd in PE respectively.

5.5 Own experience of mathematics as a pupil/student

Although few teachers referred to their initial training as an important influence, many more spoke, often with considerable emotion, of their own bad experiences of mathematics as a learner.

> *The trauma of school maths...I obviously had this ghastly teacher at school (Carole Y2)*

This appeared to relate to all levels of the secondary school, but particularly to mathematics at A-level and in one case also at university (see Chapter 4). Mathematics had been presented as an arid and meaningless subject:

> *I did pure maths A-level - in hindsight I look back on that with horror - and I did not enjoy it at all, having really enjoyed it up until then -- it was very very formal (Faith Y4)*

Some teachers contrasted the negative views that they held of mathematics at some point with their very positive views now; two of the highly effective women teachers actually used the form of words *"I love mathematics"*.

For some highly effective teachers their school experience had been an important factor in the way they chose to teach mathematics themselves:

> *It's affected the way I teach, I think, because I would never say to a child "Oh aren't you stupid" or "What a silly way to do that"...I would always, even if they are struggling, be positive about what they* <u>*can*</u> *do and try and take them on from there and not make them frightened...because I know what it can feel like, so I always try to make them enjoy it and be confident with what they do...(Carole Y2)*

5.6 Other teacher factors

As might be expected, in addition to initial training, CPD and experiences in school, in interview the teachers raised a number of factors that they considered were significant influences on their development as effective teachers. These factors included:

- predisposition
- age
- family influences
- private study
- involvement in CPD programmes for other teachers.

Predisposition
One *connectionist* teacher, Anne, expressed the view that the way she taught simply reflected her own personality, and was unaffected by her training. It is possible that this is true and that some teachers are predisposed to adopt a connectionist orientation, but an investigation of this was beyond the scope of this project.

Age
Age was not a strong factor in either effectiveness or orientation. Most of the highly effective connectionist teachers were both mature and very experienced. The way the case-study teachers were selected to include primarily the teachers perceived in

schools as the most effective meant that the case-study sample included a high proportion of more experienced teachers.

Faith was a young and highly effective *connectionist* teacher, with only three years' teaching experience. So was a highly effective Australian teacher from a validation school who had a *connectionist*. orientation. Both were teaching in inner city schools in London, which often have a high proportion of recently trained and/or overseas trained teachers.

The two strongly *discovery* orientated case study teachers were of very different ages and degrees of experience, but the two strongly *transmission* orientated teachers were both in the older and more experienced age group. It seems possible that this might be the norm for *transmission* teachers, but more data would be needed to test this.

Family influences
Both Alan and Carole referred to the changes to their thinking that had taken place as a result of helping their own children with their mathematics. In Alan's case:

> *I actually taught one of my own children ... and because I saw the home side of what was being taught at school. ... I began to see that there really was a big difference between expecting a child to see the links and giving them the skills to see the links. (Alan Y5/6)*

Alan also described conversations that he had with his brother-in-law, a professional mathematician, which had inspired him to develop his beliefs about the inter-related nature of mathematical ideas. They had worked together in developing new teaching approaches, for example to directed numbers. These interchanges obviously provided ideal but unusual opportunities for professional development in subject knowledge.

Private study
Most teachers explained that they did not find time to keep up with reading, with all the administrative as well as day-to-day teaching requirements of school. Many also had families to care for in the evenings, weekends and school holidays. Carole was probably fairly typical of the highly effective teachers in her reading:

> *...the good old Times Ed every week, especially the maths supplements, and sometimes if I see a really good book - there's a recent one come out on research on mathematics teaching (Carole Y2).*

At least two teachers had given up Open University mathematics programmes they had started because of family and professional pressures. The only time such a distance learning programme was cited as influential (by one of the effective teachers who was not easily categorised) was where it was co-ordinated by an LEA and a group of teachers met regularly in the professional centre under the leadership of an experienced advisory teacher.

However one of the other highly effective teachers, a mathematics co-ordinator, did suggest that she was a voracious reader of relevant publications. Although a few schools were corporate members of the Mathematical Association or the Association of Teachers of Mathematics and received the published journals, no teacher interviewed was an individual member.

Involvement in CPD programmes for teachers from other schools
Three of the five highly effective *connectionist* teachers mentioned that they had been involved in running CPD programmes in their LEA. This may well be a result of, rather than a cause of, becoming effective. Nevertheless it seems likely that such experience assisted teachers in formulating their own beliefs and relating these coherently to their practice. It seems likely also that experience of running CPD would enhance their ability to influence their colleagues, as in Schools A and B. For example, Anne had worked part-time as part of the county support teams in mathematics, science and languages, co-ordinating training in federations of small schools. Barbara has been released to do advisory work for the LEA in other schools and in particular to run classes for able and talented children from different schools to demonstrate good practice to their teachers. Prior to joining the school, Alan had been seconded from his post running a county field centre to serve on the science support team in the county, training teachers to introduce science at Key Stage 1.

5.7 School influences

Distribution of effective teachers between schools
It was clear that in some of the schools in the sample there was a very strong mathematics policy and an established co-ordinator providing firm leadership, both of which had the effect of spreading knowledge, beliefs and good practice between all teachers. In other schools there was considerable diversity of practice, with little more than a published mathematics scheme or a rather general scheme of work in common.

While it was not our purpose in the project to examine the effects of school factors on pupils' achievement, the effect of schools on the professional development of teachers who work in them is clearly a key factor in teachers' effectiveness. After summarising the results for the teachers in relation to the schools, some of the ways in which the most effective schools helped their teachers to develop professionally are described.

Table 5.1 shows the distribution of the Y2-Y6 teachers in each school between the categories of highly effective, effective and moderately effective.

		highly effective	effective	moderately effective
Focus schools	School A	12	1	0
	School B	4	4	2
	School C	2	0	2
	School D	1	3	5
	School E	2	5	2
	School F	2	2	5
Validation schools	School G	1	1	2
	School H	1	1	2
	School I	0	2	1
	School J	1	0	1
	School K	0	2	4

Table 5.1 The distribution of different categories of effective teachers across the schools in the study.

It was clear that School A had a quite outstanding performance, with 12 out of 13 teachers judged to be highly effective, and the thirteenth teacher only a little short of this boundary. School B also had a strong profile although with a much broader range. This suggests that it is possible for a school to be extremely effective in improving the practice and results of its teachers.

Table 5.1 shows that the remaining focus schools C, D, E and F contained a range of teachers of different levels of effectiveness, and were not therefore dissimilar to validation schools G, H and J. In each of these schools there were one or two highly effective teachers but a larger number of teachers of moderate effectiveness. School I was a village school with only three teachers and therefore was probably not essentially dissimilar from the other schools, in that the two teachers of the older age groups were effective and only the Key Stage 1 class had moderate gains.

The cumulative influence of the high effectiveness of the teachers in School A, and to a lesser extent School B, can be seen in the differential performance of these schools when pupils' results in our numeracy test are compared across the whole primary school period. Table 5.2 shows the ranking of schools in their results (mean test scores, not gains) in October of Year 2 and again towards the end of Year 6. (Schools A, G and H have the same Year 2 October mean) Similarly the cumulative effect of the low effectiveness of teachers in School K is clearly demonstrated. (Although School J, serving a very disadvantaged inner city population, remained lowest in the ranking, by the end of Year 6 it had reduced the margin separating it from the other schools.)

Entry to Year 2	Rank	End of Year 6
E	1	A
C	2	E
D	3	B
K	4	C
I	5	I
A}	6	F
G}	7	G
H}	8	D
F	9	K
B	10	H
J	11	J

Table 5.2 Ranking of schools in descending order of mean test scores (not gains) at entry to Year 2 and towards the end of Year 6

The independent schools

It is worth examining separately the performance of the three independent preparatory schools in the sample. The two independent preparatory schools included as focus schools, Schools C and E, were rather different from School K in relation to the performance of their teachers, but shared some similarities with each other. They were also in some ways similar to most of the state schools in the sample in having teachers at both extremes of effectiveness.

At the beginning of Year 2 both schools had very high results in the tests, well above those of all the state schools. Moreover, in School C, both Year 2 teachers also had very high gains. Similarly School E had highly effective teachers in Years 3 and 4. While the social and academic selectivity of these schools may have contributed to high performance, and the very small classes may have contributed to high gains, it is clear that high gains were by no means guaranteed by these favourable conditions. The only case study teacher in an independent school who was highly effective, Carole (Year 2) had a *connectionist* orientation and the same characteristics as those of most of the highly effective teachers in state schools.

Indeed the specialist mathematics teachers of the Year 5 and 6 classes in Schools C and E, Cath and Elizabeth, who were categorised as having a *transmission* orientation, had low gains, even after adjustments had been made to correct a test-bias against relatively high-scoring pupils. Thus by the end of Year 6, School C ranked fourth, having been overtaken by two state schools, Schools A and B. School E, which started as highest, had only been overtaken by School A, but School B had almost caught up.

An explanation of a tendency to move to transmission teaching in the older primary age groups in independent schools was the perceived requirements and pressure of impending common entrance examinations (some taken at age 11). This meant that numeracy was likely to be regarded as of much lower priority than mathematics examination performance in these years, and that there would not necessarily be a perceived need for consistency of approach throughout the school. Thus while these two preparatory schools were in some ways similar to most of the state schools in the sample in having teachers at both extremes of effectiveness, they were different because of evidence of changes in the orientation of teaching with age, due to increasing examination pressures. It is possible that publication of results at Key Stage 2 will generate similar transmission tendencies at Years 5 and 6 in the state system, and hence tend to reduce, rather than increase, numeracy standards.

School K, a two-form entry independent preparatory boarding school, was a validation school selected as a school thought to have reasonably sound teaching of numeracy. In the event it turned out that it had a relatively weak performance. At the beginning of Year 2 it ranked fifth out of the 11 schools, but by the end of Year 6 it came ninth. No teachers were highly effective, and not only did the majority produce low gains, but some were at or towards the lower end of this category. This may reflect the fact that there has traditionally been less emphasis on continuing professional development in the preparatory sector. It should also be noted that, as stated in Chapter 1, there was a lack of comparative value-added data on preparatory schools, which made selection of effective schools more problematic.

Factors present in effective numeracy schools

Did School A, and to a lesser extent School B, adopt some means of developing their teachers which was not present in other schools? Or were other factors responsible for the performance?

We will consider first whether other factors could have influenced the result. First it should be noted from Table 5.2 that on entry to Year 2 all the state schools in the sample had mean scores which were very close, with School A towards the lower end. (The only exception was the inner city school, School J.) By the end of Year 6, School A was scoring ahead of all other schools, including the independent schools, with School B not far behind, and the range of results was much larger.

School A admittedly devoted additional resources to mathematics in that the two classes in each year group were set into three sets for mathematics. However School D, which was much less successful, also set in an identical way to School A using extra teachers (At School A the setting had been a recent development whereas the staff in School D who had been setting for longer were dissatisfied with setting and were considering returning to mixed ability teaching by the class teacher). In School B there was no additional staffing and no setting. Similarly the published mathematics scheme used as a support in School A was used in many of the other school in the sample. Thus although it is possible that other factors played some part in the enhanced performance in School A, it seems unlikely.

School A was the only school to have two of the five case-study teachers identified as highly effective *connectionist* orientated teachers, Anne and Alan, with the third case study teacher in the school, Alice, also highly effective but only having some characteristics of this orientation. Anne was a very experienced teacher, and had been the mathematics co-ordinator for five years, with assistance in Key Stage 2 from Alan, the Deputy Head.

School B, also a very effective school but slightly less so than School A, also had one very experienced highly effective *connectionist* case-study teacher, Barbara, who had been the mathematics co-ordinator for seven years. School F was the only other school with an identified *connectionist* as co-ordinator, Faith, although she was very much younger and had been mathematics co-ordinator for only two of her three years of teaching experience. All three of these co-ordinators had been on extended mathematics CPD programmes, but Faith's had only finished in the previous term. It therefore seems possible that the strong and stable presence of established connectionist teachers particularly assists the development of other staff.

Many aspects of the co-ordination of mathematics in School A were common to those in most other schools:

- the co-ordinator ran INSET days for staff
- teachers planned their teaching working together in small groups
- there was a recently agreed scheme of work for the school
- the co-ordinator was available for consultation and for support in solving any problems.

The aspects that appeared to be different about School A were:

- strong leadership on appropriate *(connectionist)* teaching orientation, with either Alan or Anne being a full member of each Year Group team.
- discussion of teaching methods and activities at a more detailed level than seemed to be the case in other schools.
- provision of time for Alan and Anne to work alongside other teachers in their classrooms to demonstrate the approved school methods.

Alan had time available to work alongside other teachers because of his reduced timetable as Deputy Head, with three afternoons free. Anne had one morning a week in the previous year and had been given time for one term that year as part of a rotation of time to co-ordinators, paid for out of an unexpected budget surplus.)

These ways in which Anne and Alan encourage the development of the other teachers is reflected in their own accounts:

> *The way that we do our planning in school is based on groups of people working together ... it goes on a lot...I work closely with the other Year 6 teachers, when I am doing Year 6 teaching, and I work closely with Year 5*

teachers when I am doing Year 5 teaching, but Anne, who is the maths co-ordinator, and myself have discussed the maths curriculum, for example. So it takes place on a number of levels....It's our maths policy to move away from them seeing maths as a book based activity to being something that impinges on real life...Everybody is aware of it and moving towards it...In year 6 all of the maths sets are around about now looking at probability...(Alan then gives an account of how his colleague, Alice, is tackling it, and explains that this way is slightly different from his because of their slightly different approaches)...I have been teaching in classes with teachers and have been doing some training where I have been teaching lessons and they have been taking notes. (Alan Y5/6)

I work closely with each (year group) team so I talk about the work I do and they talk about the work they do. Then we try to pass things around and we have a lot of discussion about the problems that children have, how we can solve them, and I will look for things to support them. I am trying to encourage people to try different things...Last year I did co-ordinators' time, for one morning a week I would go to different parts of the school and I would go and work with the teachers in the classrooms for maths ...they all had things they wanted me to do ...I make sure I am fulfilling their request, but I am also showing them ...I will try to do it in different ways (Anne then illustrates this, talking about devising an activity on addition which 'threw the teachers' as it challenged their assumptions about what pupils could do, but was sensibly tackled by 5 year olds) (Anne Y2/3/4)

It is clear that this approach provides very supportive in-school professional development, as one of the other highly effective teachers in School A describes:

My training has been really within this school (she has not been on any external courses). We do have INSET days, curriculum meetings, planning days. I talk in detail with my two year partners...the three of us would talk about actual progress and any planning, everything to do with our work in maths and we plan together and also after, we talk about how well it went ... if I had made the planning too formal so perhaps the children could not cope with something during the week ..we discuss it at that level. (Alice Y6)

This in-school approach also assists in inducting new teachers into the school mathematical ethos:

Last year I had a new teacher who joined the team and I was able to work really closely. I felt that when she first came she was a 'sums' person, she liked to do the sums , but she responded really well. (Anne Y2/3/4)

Several features of the school that cannot necessarily be shared by other schools would appear to enable this approach to be effective:

- an experienced and confident co-ordinator and deputy head who share an approach to teaching mathematics
- a head who encourages close monitoring of pupil progress and has a particular interest in mathematics
- relatively stable staff
- a large school which enables some flexibility in staffing.

School B was less well-favoured, with Barbara combining the posts of Deputy Head, in charge of both curriculum and staff development, with those of co-ordinator for both mathematics and assessment. Nevertheless this did give her some time to work alongside other colleagues. School B also had some school policies that helped to promulgate Barbara's *connectionist* orientation, for example the use of focus groups:

> *I try to make sure that I sit with each focus group once a week and generally I try and make notes on the children - part of our teaching and learning policy. (Brian, Y2).*

As in Schools A and B, Carole, the *connectionist* teacher in the independent sector, reported the significance for professional development of being able to discuss methods of mathematics teaching in detail with other teachers of the same year group:

> *My parallel teacher and I talk a lot...and it's being dissatisfied with the way we felt things were going that this year we both started changing ... it seems to be working much better...we plan a lot together ..we don't do every lesson the same but we try and keep them more or less together. (Carole Y2)*

It seems likely to have been this factor of discussion with a colleague who happened to be like-minded that enabled both teachers to produce very high gains in the tests, even though the mathematics leadership in the school did not correspond to their views.

5.8 Summary and discussion

The data presented in this chapter suggests that there is no unique way of arriving at effectiveness in the teaching of numeracy. However there clearly are some routes that commonly seemed to be significant in leading to effectiveness.

- Teachers who did not themselves have strong *connectionist* orientations did become highly effective in a school where *connectionist* beliefs are pervasive.
- Extended CPD mathematics education programmes (for example, 20-day GEST programmes) were perceived by teachers as highly influential in developing their beliefs and practice. Interviewing pupils, discussing different mathematical strategies for mental calculation and more generally for problem-solving, and appreciating research findings were all cited as

key ingredients of such programmes which encourage teachers to reflect on their own beliefs and practice.

- Short programmes and extended non-mathematics programmes had little effect on beliefs or practice in teaching numeracy. There were no examples of distance-learning programmes being influential except in the context of an LEA co-ordinated group with expert leadership.
- Initial training was perceived to have little influence on effectiveness, in some cases because of its distance in the past, but in other cases because of the severe shortage of time available for mathematics, especially on primary 1-year PGCE programmes.
- Teachers' own negative attitudes to mathematics as a result of their experiences as a learner of mathematics could be changed during CPD. For some teachers, their own negative experiences as learners influenced them in trying to make mathematics enjoyable and accessible for all children, and was sometimes associated with a high degree of effectiveness.
- Neither age nor experience were strong factors in either effectiveness or orientation, except that there may be a tendency for *transmission* orientated teachers to be older and more experienced than average.
- Schools in which there was a clear *connectionist* philosophy had a significant effect on raising standards by enabling curriculum leaders to work closely with other teachers. This happened both through planning teaching approaches and in working together in the classroom.
- A range of outside-school activities and ongoing discussions played a part in changing teachers' beliefs about teaching and learning mathematics. Participating in advisory work or in leading CPD programmes was beneficial.

In examining these findings, three common threads seem to run through them, *attitudes to mathematics*, *time* and *talk* .

Attitudes to mathematics
As discussed in chapters 3 and 4, a characteristic of highly effective, *connectionist* orientated teachers lay in their beliefs about the nature of mathematics rather than in their content knowledge of mathematical concepts and procedures.

However most of the *connectionist* teachers reported that they had not always felt like this. As a result of arid teaching at school, aiming at mastery of techniques rather than at meanings and applications, they had previously experienced feelings about mathematics ranging from boredom to dislike and even trauma. A-level mathematics programmes were mentioned frequently, but some teachers cited earlier phases in secondary schools or university mathematics degree programmes.

While these highly effective teachers had generally, but not universally, overcome these negative views to become confident and *connectionist* in their views of mathematics, it is clear that other teachers, especially the *transmission*-orientated teachers, still saw mathematics as a fragmented set of techniques and knowledge to be memorised. Thus the cycle of negative attitudes was likely to be reproduced in future generations of pupils.

Teachers' reports suggest that *transmission* orientations are likely to be much more common among secondary, A-level and possibly HE teachers. This is supported, for example, by the results of the recent Third International Mathematics and Science Study (TIMSS), where lower proportions of English secondary mathematics teachers than those in almost any other country thought that creativity was important in mathematics; correspondingly a moderately high proportion, over 45%, of English 13-year-olds thought that memorising their textbook was a good way to become good at mathematics (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996). The fact that mathematics is perceived to be 'hard and boring' is also well-known to be a factor in low take-up of A-level mathematics (Department for Education, 1994). It is possible that additional pressure to raise examination results in the state system will have the same effect as common entrance seems to have in the independent sector, which is to increase the extent of *transmission* teaching.

Extended CPD programmes were perceived by teachers to be a successful way of changing their views of the subject, while at the same time kindling their enthusiasm for and confidence in mathematics. Even highly effective teachers who had not been on such programmes had less positive attitudes than those who had.

Clearly any initiative which results in more positive and more holistic attitudes to mathematics in secondary schools, A-level programmes and higher education will also in the long term pay dividends in increasing the numbers of primary teachers with *connectionist* attitudes. Such initiatives have been shown to be successful, particularly among girls (Black, Boaler, Brown, Murray, & Rhodes, 1996). They would seem to be necessary if we are to break into the cycle of narrow and negative attitudes and of low effectiveness in the teaching and learning of mathematics.

Time

The study has confirmed the already well-known fact (e.g. Joyce & Showers, 1980) , that short CPD programmes have little effect on teachers' beliefs and practices, but that there does seem to be a significant effect from programmes extended over a long term, such as a 20-day GEST programme, in which teachers get time to reflect and reconsider.

Lack of time for considering mathematics teaching and learning was also a criticism of 1-year PGCE primary programmes and would seem to be one reason why initial training was not put forward as a critical influence.

Time was also a factor in the highly effective school, in that the arrangements in this school had been stable over a relatively long period. This contrasted with the inner city school that had seen high turnover of co-ordinators and other teachers. Other schools gave signs of moving in this direction of spreading good practice from a strong co-ordinator, but progress was slow. Generally, findings here support other evidence that changes in teachers' beliefs and practices are not achieved overnight (e.g. Fullan, 1991).

Time seems to be important in the day-to-day as well as in the long-term picture. For example the two highly effective mathematics curriculum leaders in the strong school were able to obtain and utilise time to spend with other teachers both in class and in planning groups. Administrative responsibilities and other pressures on the time of the most effective teachers in other schools was constraining their ability to interact with other teachers about mathematics. It also seemed important that the reasons for teachers not keeping up with reading, or for giving up in the middle of distance learning programmes, seemed to be pressures on their time, both excessive professional demands and legitimate family pressures on leisure time.

Talk

The most effective use of time seemed to be in productive talk. One significant form of talk that had changed teachers' views about teaching was a discussion with single or small groups of pupils or with other teachers with the aim of investigating different mental strategies. This was as a focused part of a mathematics education programme, but it had encouraged these teachers to spend longer listening to children in their classrooms. In some cases the conversations had been at second hand, through reports of research and videos.

More general references were also made to the ability to discuss with other teachers specific details of the way particular strategies or ideas could be best introduced to children. This focused discussion took place, generally in the presence of someone with a leadership role, and with teachers or other people with whom it was possible to feel at ease. Talk, which was reported to have had a significant influence, occurred:

- on extended programmes outside school, organised by either LEAs or HE institutions, with opportunity to meet other teachers;
- in year group planning meetings;
- before and after a co-ordinator had taught alongside them in their own class;
- outside school with someone (e.g. a relative) with a professional knowledge of mathematics teaching.

However moderately effective teachers also met to plan, with little obvious developmental effect, which suggests that it is important that at least one member of the group, generally the leader, already had a *connectionist* orientation.

There is OECD evidence that English teachers have longer teaching hours and less free time for discussion than those in other countries. Moreover when they do meet, they spend much less time in discussing teaching methods and much more time on administrative and assessment matters than teachers elsewhere (Bierhoff & Prais, 1995).

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Chapter 6 Implications

6.1 Personal factors affecting teaching effectiveness

Orientations (Chapter 3)
The three teaching orientations described in Chapter 3, *connectionist*, *transmission* and *discovery*, provide some insight into why some teachers are more effective in teaching numeracy than others, and into how teachers can become more effective.

The mathematical and pedagogical purposes behind particular classroom practices seem likely to be more important than the practices themselves in determining effectiveness. For example, our highly effective, connectionist orientated teachers demonstrated a range of classroom organisation styles including whole class teaching, individual and group work. On such measures their practices were indistinguishable from those of the teachers who were only moderately effective.

There was some support for the view that beliefs and practices may develop in tandem.

Questions:

- *Should there be some shift in national and local priorities from affecting practices to increasing teachers' awareness of beliefs in order to affect learning* outcomes?
- *Could materials be developed out of this study to assist teachers to examine their beliefs in relation to their practices, using information about the three 'ideal types'?*

Knowledge of Mathematics (Chapter 4)
What would appear to matter in relation to the effectiveness of teachers is not formal qualifications or the amount of formal subject knowledge, but the *nature of the knowledge about the subject* that teachers have. The connectedness of teachers' mathematical knowledge in terms of their appreciation of the multi-faceted nature of mathematical meanings does appear to be a factor associated with greater pupil learning gains.

This suggests that 'more' is not necessarily 'better' in terms of the mathematical subject knowledge that teachers' need to help pupils develop their understanding of mathematics. In particular, primary school teachers may need to develop a fuller, deeper understanding of the number system and number operations and relations, and the way different representations of these interconnect, in order to effectively teach numeracy. Chapter 4 includes some suggestions for aspects of mathematics that teachers might benefit from concentrating on.

Questions:

- *How can teachers best be helped and encouraged to deepen their understanding of numeracy connections?*
- *Can school or university mathematics courses such as A-level and degree programmes be re-designed so as to assist in the longer term?*
- *To what extent will ITT be able to contribute, or must CPD always be the major instrument?*
- *Are new materials needed, for example, guidance on how to carry out concept mapping, as used in this project, as a self assessment tool?*

Knowledge of pupils (Chapter 4)

The highly effective, strongly *connectionist* teachers paid much attention to pupils' understandings. They documented carefully information about pupils' learning and then used it to inform and develop their teaching.

In contrast some of the teachers who were only moderately effective used assessment simply to check up on how much of their own teaching had been learnt; any gaps in understanding were dealt with through re-teaching and additional practice.

It would seem that teachers should be encouraged to review their assessment practices at both the class and school level and examine the extent to which assessment is genuinely used to inform teaching. Experiences that teachers had which asked them to listen to pupils' methods of mental calculation appeared to be a very effective method of raising their awareness of the importance of assessing pupils' strategies rather than solely relying on marking based only on whether or not pupils arrived at correct answers.

Questions:

- *How can teachers be encouraged to review their assessment and record keeping strategies?*
- *Would it help to have examples of systems that are used by connectionist teachers and are manageable, but allow assessment and recording of information not normally documented (for example, pupils mental strategies)?*
- *To what extent is this useful unless teachers embrace connectionist beliefs?*

Teachers' continuing professional development (Chapter 5)

Extended CPD mathematics education programmes (for example, 20-day GEST programmes) were perceived by teachers as highly influential in developing their beliefs and practice. Short programmes and extended non-mathematics programmes had little effect on beliefs or practice in teaching numeracy. There were no examples of distance-learning programmes being influential except in the context of an LEA co-ordinated group with expert leadership.

Initial training was perceived to have had little influence, in some cases because of the severe shortage of time available for mathematics, especially on primary PGCE programmes.

Questions:

- *Should extended programmes of CPD in mathematics education have priority, and if so who should provide them and what components should they contain?*
- *Would it be useful if they included aspects which highly effective teachers in this study perceived to be helpful i.e.*

  - interviewing pupils and reporting on their mental strategies;

  - detailed discussions about the different ways in which specific numeracy ideas can be presented, within a wider view of the subject and its pedagogy;

  - use of research data and videotapes, and research finding;

  - working on mathematical problems at teachers' own level;

  - discussions about significant themes in mathematics and exploring connections?
- *Should additional time for numeracy be found in initial training? Can this be done within a 1-year PGCE or is additional time needed?*
- *Should the contents of ITT include some or all of the features in the list, or are they better left for CPD?*

6.2 School factors affecting effectiveness

The main focus of this research was on the individual teacher and the research was not designed to examine in depth the impact of school policy and practices on individual teachers. However, some aspect of schools did emerge as influential and are worth noting.

The data provides some evidence that teachers who did not themselves have strong *connectionist* orientations could become highly effective in a school where *connectionist* beliefs are pervasive.

Schools in which there was a clear connectionist philosophy had a significant effect on raising standards by enabling curriculum leaders to work closely with other teachers. This happened both through planning teaching approaches and in working together in the classroom. Time and stability were important ingredients here.

Questions:

- *How can effective teachers of numeracy already in school be better identified? What are the best methods of enabling their expertise to be spread among other teachers?*
- *Is more time for co-ordinators to work alongside other teachers necessary, and if so how can the necessary financial support be found?*
- *Where there are no highly effective teachers of numeracy in a school, is it possible to develop staff already there, and if so will extended CPD be sufficient?*
- *How can staffing become more stable so as to allow time for in-school development, especially in inner city schools?*
- *What is the role of the LEA in assisting the development of a school in numeracy, and in assisting the professional development of individual teachers in this area?*
- *Do LEAs currently have the staff expertise, and staff time, to fulfil this role? What mechanisms are appropriate for independent and grant-maintained schools?*

6.3 Recommendations for further research

The project has also identified several areas that would benefit from further research. These include:

- refining and further validating the model of three types of teaching orientation
- exploring the nature of 'connected' knowledge in more detail, for example focusing on place value, or fractions, decimals and percentages
- exploring changes in teachers' beliefs over time, including the role of different elements in the change process
- examining the take up of CPD in mathematics - who participates and why?

Appendix 1.1. Characteristics of the Focus Schools

Classes in sample

School	Type	Locale	Form entry	Pupils on roll	Classes in sample	Teachers in sample
A	primary	School A is located in the pleasant suburbs of a large town.	2	427	18	15
B	primary including nursery	School B is located on the outskirts of a major conurbation in an area which has a mixture of council and private housing.	2	482	12	12
C	prep. and pre-prep.	School C is located in an attractive Home Counties town. The school has a wide catchment area.	2	212	8	6
D	primary	School D is located on the outskirts of a large conurbation. The local housing varies from large detached properties to smaller private and council houses.	2	414	13	13
E	prep. and pre-prep.	This school occupies an attractive rural location in the Home Counties. Pupils are drawn from a wide catchment area.	3	449	18	13
F	primary including nursery	School F is an located in a densely populated inner city area which has a wide range of housing stock. It is one of several primary schools in the area.	2	438	11	11

Appendix 1.2. Characteristics of the Validation Schools

School	Type	Locale	Form entry	Pupils on roll	Classes in sample	Teachers in sample
G	primary	This school is located on the outskirts of a small village in the Home Counties.	1	125	6	5
H	primary including nursery	School H is situated on the outskirts of a large conurbation. It is located in the midst of a council housing estate but draws its pupils from a wider area.	1	233	5	5
I	primary	School I is situated in a small attractive village in the Home Counties.	0.5	85	3	3
J	primary	This inner city school is located on a busy urban road. High rise flats and commercial buildings dominate the surrounding area.	1	198	3	3
K	prep. and pre-prep.	This school is situated in extensive grounds in an attractive rural location. Pupils are drawn from a wide area. A proportion of the pupils board.	2	228	9	7

Appendix 1.3 Development of pupil tests

Introduction & Aims

This appendix describes the design and use of a test of numeracy which was developed as a research instrument as part of a project attempting to identify and to characterise effective teachers of primary numeracy The test was required to assess the progress in numeracy over a period of six months of children in each of the age groups between 5 and 11.

Features of its design depended on the role that it was required to perform, which will be described under the three headings of:

- requirements of the sample;
- validity as a measure of numeracy;
- logistic constraints.

Design: the Requirements of the Sample

The basic requirement was to design an instrument that would measure the 'value-added' numeracy levels that were contributed by each of a sample of approximately 100 teachers, constituting all teachers of children from Year 1(age 5 to 6 years) up to Year 6 (age 10 to 11 years) in 12 primary schools. The primary schools included two from each of three local education authorities and two independent (private) schools, all of which were identified by overall school results in local mathematics tests to have 'high added-value'. The remaining four schools included one from each area to act as a control of a defined type. The schools were from diverse social backgrounds, and included both a small rural school with children of different ages in one class, and a large inner city school with several different classes in each year group. Some of the schools with high 'added-value' were nevertheless expected to have initial low attainment because of the nature of their intake and environment.

With regard to the sample size, using the distributions of attainment found by Wiliam (1992), it was estimated that a pool of 24 teachers teaching classes of 30 pupils would give statistically significant results provided the improvement in pupil attainment produced by the more effective teachers was equivalent to an effect size of 0.1, judged in this context to be a reasonable figure. A sample of 100 teachers was thus well above this minimum size.

The size of the pupil sample was expected to be about 2500 pupils, but spread fairly evenly across the five year groups from Year 1 to Year 6, giving 400-500 in each year-group.

Clearly it would not be feasible to have a single instrument across the ages from 5 to 11 if we were to do justice to the progress made by both low-attaining younger and high-attaining older children. Nevertheless since other work at King's (summarised in Wiliam, 1992) has demonstrated that some 7-year-olds have higher mathematical attainment than some 13-year-olds in mainstream schools, it was felt important to have as many common items as possible across the age groups.

In the end it was decided to have three tiered test-instruments with similar formats: Tier One for Years 1 & 2 (ages 5-7), Tier Two for Years 3 & 4 (ages 7-9) and Tier Three for Years 5 & 6 (ages 9-11). The number of common items is as shown in Table 1:

	Items in T1 only	Items in T1 & T2 only	Items in T1&T2 &T3	Items in T2 only	Items in T2 &T3 only	Items in T3 only	TOTAL ITEMS
Tier 1 (5-7yrs)	28	28	31				87
Tier 2 (7-9yrs)		28	31	6	62		127
Tier 3 (9-11yr)			31		62	51	144
All Tiers	30	28	31	6	62	51	208

Table 1: Constitution of the three test tiers, indicating common items

Where different 'tiers' of tests are needed to ensure that the tests administered are appropriate for the pupils with whom they are used, as in this case, the different tiers can be equated using standard procedures (Holland and Rubin, 1982) so that the scores obtained on one tier will be compared directly with those from the other two.

A further consequence of the desire for common formats across the whole age range was the decision to favour an aural test administered by teachers to the whole class at once. It was considered that there would be too many weak readers among younger pupils to allow a purely written test to act as a valid measure. The aural format was also arrived at from a consideration of the requirements with respect to content, as described below.

Design: The Validity of the Test as Measure of Numeracy

A broad definition of numeracy was chosen by the project:

> *Numeracy is the ability to process, communicate, and interpret numerical information in a variety of contexts.*

Clearly in a test to be administered on a relatively large scale with limited resources for administration and marking, it would not be possible to assess some aspects of this, e.g. oral communication in a variety of genuinely realistic contexts. Nevertheless an attempt was made to include contextual as well as purely numerical items, and to cover application and conceptual as well as procedural skills. An investigatory item is included in order to assess the ability to apply systematic processes and mathematical reasoning in a numerical context.

Because of the aim to assess the degree of sophistication of the mental strategies that children had available, rather than routine written procedures or simply the ability to obtain a correct answer by any means including tortuous primitive counting, it was decided to include in the tiered tests only aural items, with short response times. This decision was also in line with the requirements of having a common format even with comparatively young children who might be poor readers, as described earlier.

The basis of the design of the tiered tests was an aurally administered diagnostic numeracy test for whole classes aged 7-11 that had previously been developed at King's by Brenda Denvir (Denvir & Brown, 1987). This was itself adapted from a diagnostic interview that was the product of earlier research to identify a hierarchy of development of numeracy skills in low-attaining 7-9 year-olds (Denvir & Brown, 1986 a,b). The aural test had been validated against the interviews, and was found to be very reliable in the sense that items correct in the test almost always (97.5%) implied corresponding items would be correct in the interview. Although the agreement was less good in the opposite direction (86% of the items correct in the interview were also correct in the class test), this was judged reasonable as children were expected to be more motivated by the presence of an interviewer. The main concern was that about 15% of pupils significantly underachieved in the test as compared with the interviews.

The test was supplemented by items developed in the same institution for secondary children and published as the Chelsea Diagnostic Tests (Hart et al., 1985), by items designed to assess additional aspects of the newly re-written English National Curriculum (Department for Education, 1995), and by items assessing aspects referred to in other frameworks e.g. McIntosh, Reys & Reys(1992).

The skills assessed were:

Understanding of the number system
- *Knowledge of the standard number word sequence*
- *Interpolation between numbers on a number line, with both whole number intervals and fractional intervals*
- *Knowledge of which numbers are ten more and ten less , a hundred more and a hundred less than a given number*
- *Effect of multiplying by 10 and identification of number of 10s in a number*
- *Ordering of negative numbers in context*
- *Identification and ordering of fractions*

Methods of computation
- *Knowledge of addition and subtraction bonds*
- *Mental addition and subtraction of larger numbers*
- *Knowledge of multiplication bonds*
- *Multiplication and division: enumeration of items grouped in twos, fives, tens and hundreds (including representation to two decimal places)*
- *Use and calculation of percentages*
- *Calculation using ratios*

Solving numerical problems
- *Solution of complex addition, subtraction, multiplication and division word problems*
- *Solution of problems involving money*
- *Representation of complex word problems*
- *Identification of representation of complex word problems*
- *Appreciation of relationships between numbers and operations*

The next section illustrates part of the test design framework, including examples of items, correspondence with the National Curriculum level descriptions and with item numbers to corresponding questions across the three tiers.

Part of design framework for tiered tests

Skill	NC level	Year 1/2	Year 3/4	Year 5/6	Example	Aspects of number knowledge
Solution of complex addition and subtraction word problems					Twenty two people are on a bus, thirteen are children. How many are adults?	Appreciates semantic relationships in additive word problems, using addition and subtraction
• numbers up to 10	2	**B5a** B5c	**8a**			
• numbers up to 20	3	B5b **B5d**	**8b** **8e**	**8a** **8d**		
• two-digit numbers	4		**8c** **8d**	**8b** **8c**		
Solving problems involving money	2	B2a B2b				Addition and subtraction of amounts
Ordering negative numbers in context	4?		**8f**	**8f**	Given three temperatures, which was the coldest day?	Application of negative numbers
Representation of complex word problems					Which keys do you press on the calculator to work out ... ?	Application of four rules; formal representation of solution methods; identifying appropriate methods
• addition and subtraction	3?		**11b** 11c **11e** **11f**	**10b** **10c** **10d** 10h		
• multiplication and division	3?		**11d** **11g**	**10e** 10f 10g 10i		
Selection of representation of complex word problems					How many different sandwiches can be made from six different fillings and three types of bread?	Application of four rules; formal representation of solution methods; identifying appropriate methods
• addition and subtraction	3?	**B7a** **B7b** **B7d**	**12a** **12b** **12d** **12f**	**12a** **12b** **12d** **12f**		
• multiplication and division	3?	**B7c**	**12c** **12e** **12g**	**12c** **12e** **12g**		
Appreciation of relationships between numbers and operations	4?	A10a **A10b** A10c	**12b** **12c** **12d** **12e** **12f**	**11b** **11c** **11d** **11e** **11f** 11g 11h	Given 86 + 57 = 143 can they quickly work out 860 + 570?	

Design: Logistic Constraints

In order to be sensitive enough to assess gains over a period as short as 6 months, the tiered tests needed to be fairly 'fine-grained' i.e. required a fairly large number of similar items with a low difficulty gradient. This meant that within a single type of question there were several parts, with a gentle progression in terms of the complexity of the numbers or the operations.

For example, on one item children were asked to write down a given number, and then to write down the number that was one less than that number:

Tier 1 was given the initial numbers 30, 76 and 174;

Tier 2 was given the initial numbers 30, 76, 200 and 1100;

Tier 3 was given the initial numbers 76, 200, 1100 and 6000.

Similarly, on another item, it was written in the answer booklets that 86 + 57 = 143. Children were asked to use this to work out quickly the answers to other questions:

Tier 1 was asked about 87 + 57, 86 + 56, 87 + 56;

Tier 2 was asked about 86 + 56, 57 + 86, 860 + 570, 85 + 57, 143 - 86;

Tier 3 was asked about 86 + 56, 57 + 86, 860 + 570, 85 + 57, 143 - 86, 86 + 86 + 57 + 57, 85 + 58

Because of time constraints this meant that the number of types of item had to be severely restricted. Even so, the tests needed to contain a large number of items parts (between 89 and 144 for the different tiers).

With aural items there was a problem with maintaining concentration of both teacher and children for long periods; hence each of the tiered tests was divided into two parts to be administered separately.

The constraints of the project, which was funded for only 15 months, meant that it was not possible to carry out extended trialling of the test. This meant that most of the items had to be drawn from those on which we already had extensive data. Unfortunately this meant that some interesting item types untrialled in England, including some being used by Reys and Reys (Macintosh, Reys & Reys, 1992), were not in the end included.

The first phase of trialling was to gauge the difficulty of the items over a wider span of ages, as well as re-checking on their feasibility, since for many items it was ten years since the previous trials. The format of the answer book was also trialled since this had been re-designed.

Printed booklets were used for the responses in preference to coding sheets although this involved an extra step of coding for analysis. This was because it was felt it was very important that items should be in open response and not multiple choice format, allowing in some cases the child's own words.

Teachers were issued with posters to use for some items where it was necessary that the class should have limited time-exposure; overhead projector transparencies would have been preferable but not all teachers had ready access to these. Teachers were advised that on most questions they could repeat the instructions.

Constraints on time and funding for marking led to the preliminary trials taking place in just two schools with, in each of the six age groups in the school, three children selected as high-attaining, three as average and three selected as low-attaining. This allowed the researcher to closely observe and if necessary interact with the nine children in each group in each school tested on a single occasion together. The results nevertheless gave a fair indication of whether the items were at an appropriate level. An earlier decision only to use the test with Year 1 pupils in the second half of the school year, when most pupils would be age 6 rather than 5, was confirmed, although one school decided it wished to administer it at both times with its two year 1 classes. (Although this meant that 'value-added' data would be available for Year 1 teachers, most schools in the sample had base-line data on school entry for comparison.)

After further adjustment and reduction in the number of items, the teachers' instructions were written and the tests and instructions were trialled with teachers of each of the six year groups in a single school administering the tests. Feedback from the teachers led to further changes before the tests were mailed to the twelve schools in the sample.

The tests were carried out for the first time during October, 1995, and were repeated at the end of March or in early April, 1996. Because of the role of the test as a research instrument, no systematic evaluation took place after the first administration. However comments were volunteered by teachers either when they returned the scripts or on other occasions when their schools were visited by members of the research team. Some teachers reported that they had taken a while to come to grips with the instructions, and that the tests themselves had taken a long time to administer. Some reported that pupils found the aural format unfamiliar since much of the mathematics in the school was taught by written materials. Some were concerned that low-attaining pupils had found many items difficult, although all had achieved some correct answers. However a small number of teachers volunteered that the test fitted in very well with their ways of working.

With the exception of one school which had earlier had to withdraw from the project, the tests were completed and returned by the whole set of 90 teachers in Years 2 to 6. Feedback on the second administration suggested that teachers found it much easier to administer the second time. The format seemed also to be better received, perhaps because it had recently been announced that a test of mental arithmetic would be a

Collection and Analysis of Data

The test scripts were returned to King's College where they were marked and entered on coding sheets by a team of student teachers who had been specially trained. A researcher was on hand to answer any queries that arose. Coding sheets were scanned using an optical mark reader. Data was then 'cleaned'.

Statistical analysis was carried out using a variety of software packages, including DataDesk, Systat and SPSS, all of which allow easy importing and exporting of data in tab-delimited format.

The analysis was completed during Summer 1996. An example of the evaluation of the results on the Year 1 and Year 2 test in the first administration is shown below.

The three variables (the proportion of students answering an item correctly, incorrectly, or not answering it at all) can be interpreted as proportions, so the need for a 3-dimensional representation is avoided as they can be represented in two dimensions by a 'ternary chart'.

A ternary chart is based on an equilateral triangle, and represents each of the three variables by the distance from one of the three sides[1]. So in figure 1, the proportion of items correct is indicated by the distance of the point from the horizontal axis (i.e. points higher up the diagram were answered correctly by a greater proportion of pupils).

The point marked A in figure 1 represents an item that was answered correctly by 40% of the pupils to whom it was given, answered incorrectly by 21% of the pupils to whom it was given, and 39% of the pupils to whom the items was given made no response. This item is in fact item 0506, which was answered correctly, incorrectly, and not answered by 132, 71 and 129 pupils respectively. In contrast, the item represented by B in figure 1 (item 5102) was answered correctly by approximately 17% of pupils to whom it was given, answered incorrectly by 9% of the pupils to whom it was given, and 74% of the pupils to whom the items was given made no response.

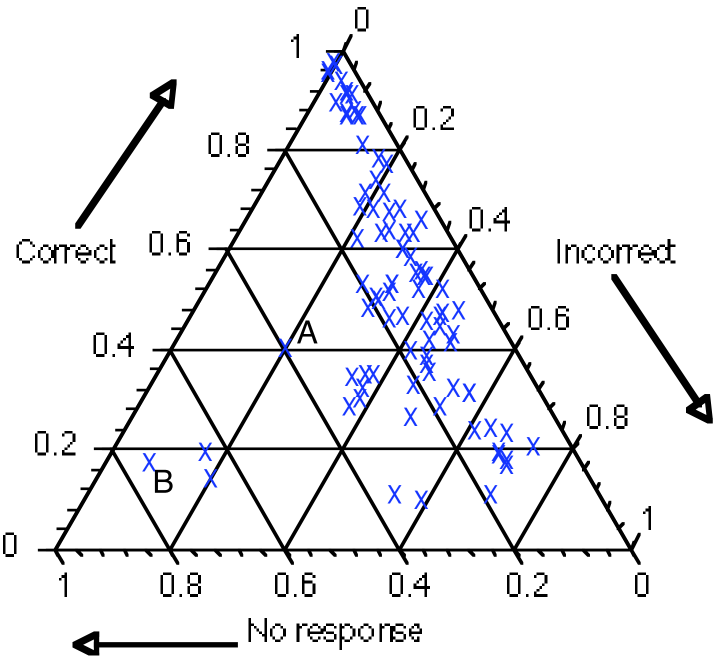

Figure 1: Ternary chart of proportion correct, incorrect and no-response in Y1&2 sample

From figure 1, it can be seen that the item facilities are widely spread from just under 10% to almost 100%, and the distribution of item facilities is quite uniform. Furthermore, for the vast majority of items, the proportion of students failing to give a response is well under 20%, with only three items having non-response rates over 40%. Since these three items (5101, 5102, 5103) all come from the same question, with the same stem, it suggests that pupils answering this item were put off by the phrasing of the item stem.

[1] The ternary chart is based on the fact that wherever a point is placed inside an equilateral triangle, the sum of the distances to the three sides is always the same.

Adjustments to the gains

Because some setted or selective school classes scored a relatively high proportion of the marks in the first administration, it was decided to use as a measure of teacher effectiveness not the absolute gains in mean class scores but an adjustment which allowed the gain to be judged as a proportion of the greatest possible gain which could be made.

Two possible formulae were tried out:

i) Adjusted gain = $\dfrac{b - a}{(T - a)}$

(ii) Adjusted gain = $\dfrac{b - a}{a(T - a)}$

In these formulae, a represents the score in the initial administration, b represents the score in the second administration, and T represents the maximum score on the test.

The effect of the first adjustment was to depress the gains of the low scoring classes by what was judged to be too great a margin, so that the second adjustment was adopted.

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Appendix 2.1: Teacher research instruments and outcome data

Five methods provided data on the teachers: questionnaire, classroom observation, interviews, concept mapping and personal construct elicitation. Each of these interconnected and while having different foci data from one source fed into the others allowing for the expansion and validation of findings.

Questionnaire

This provided background data on:

- organisation and planning for mathematics teaching
- resources and classroom materials
- training and continuing professional development

and a certain amount of evidence on:

- teaching styles
- knowledge and beliefs regarding numeracy
- beliefs on teaching, learning and assessing mathematics in general

Observation

Data was gathered from participant observation in classrooms. Approaches included a mixture of schedules structured to minimise inappropriate observer inferences and the development of detailed accounts of the flow, content and context of lessons (thick description). As significant features of practice were identified, some classroom episodes were be audio-taped and selectively transcribed to permit more detailed levels of analysis.

Existing research into effective teaching of mathematics framed initial observations. For example, the attention that teachers pay to providing instruction and learning opportunities in strategic thinking (as opposed to relying on counting strategies) was one focus of attention.

Following on from this, data collection and analysis were interactive. This 'constant comparative' (Strauss, 1987)interplay of observation and analysis enabled the theoretical sampling of lessons and rapid progressing focusing of observations.

Data collected included:

- organisational and management strategies - how time on task was maximised; catering for collective and individual needs, coping with range of attainment
- teaching styles - intervention strategies, questioning styles, quality of explanations, assessment of attainment and understanding, handling pupil errors
- learning opportunities - sources of activities, range of tasks, resources available, expected outcomes
- pupil responses - ways of working, evidence of understanding.

Interviews

Teacher interviews Semi-structured interviews produced data at different levels of generality including:

- further details on training and development; pedagogic and numeracy content knowledge; pupil knowledge and beliefs; understanding of teaching numeracy
- exploration of beliefs and good practice in teaching numeracy
- mathematical awareness of interconnection of ideas.
- exploration of critical incidents related to the above
- teachers own perceptions of what has made them successful teachers of numeracy and reasons for factors identified.
- specific details related to numeracy arising out of the observations - significant incidents as identified both by researcher and teacher
- aims and planning
- differences between teachers' self perceptions and observations

Head teacher interviews Semi-structured interviews probed issues arising out the refinement of teacher data, including:

- school policy, management and approaches to CPD as related to the research
- head's perceptions of teacher's confidence, ability and approaches to teaching.

Concept mapping

Some research literature suggests that less effective teaching is the result of lack of content knowledge, but there is little to support the conclusion that improved subject knowledge will lead to better teaching. We believed that exploring teacher's own understanding of the mathematical concepts involved in being numerate is a central focus of the research.

Traditional models of exploring levels of mathematical understanding were inappropriate, not least in the static nature of the knowledge assessed. We believe that the interconnectedness of mathematical ideas and teachers' appreciation of these are important issues. The tool of 'concept mapping' elicits detailed data on such understanding. Working one to one, the teacher generated a series of terms that they perceived as significant in being numerate. The activity continued with the teacher drawing up a network of connections through the terms. The discussion that accompanies this allowed the researcher to probe understanding and build up a rich set of data on mathematical understanding. Alongside this, the accompanying discussion also ranged over when and how the teacher came to such understandings thus informing the dissemination focus of the research.

Personal constructs

Alongside mathematical subject knowledge and understanding we also considered it important to probe how teachers make sense of pupils' learning, both in general and for individuals. The technique of eliciting personal constructs allowed for this.

This technique involved working with a set of elements: pupils' names. Presented in groups of three, the teacher identified some way in which two of the pupils displayed behaviours in their understanding of numeracy that the teacher considered to be significantly different from the third pupil. The process is repeated both with the same triad and with different triads until no new 'constructs' are forthcoming.

A set of polarised constructs was thus elicited - providing data on the models of stages of significant pupil behaviours that the teacher works with. Where time allowed, further examination of these constructs, in a way similar to concept mapping, allowed for exploration of the interconnectedness of the ideas. Finally the teacher located each pupil in the class in terms of the constructs, allowing for discussion of the significance and strategies of assessment.

As before, the requirements of dissemination were kept in mind and whenever possible, teachers probed about the origins of the understandings and beliefs being elicited.

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Appendix A: Overview of existing findings and their use in informing the study

[The following is adapted from the original submission to the TTA and provides the literature review that informed the design of the project.]

1.2.1 The National and International Context

There is currently much concern about national standards in the understanding and use of number ideas and skills. This arises from comparatively poor performance in this area, both in relation to other countries and to our own national performance in other areas of mathematics (Robitaille & Garden, 1988; Lapointe, Mead, & Askew, 1992; Bierhoff & Prais, 1995). While the concern is at all levels from early years to undergraduate intake, Ofsted (1994) focus on KS2, reporting that:

> *'In Key Stage 2, mathematics is judged to be the weakest subject in the curriculum...Pupils' understanding of mathematics is judged to be particularly weak in half of all schools... Teachers have little theoretical understanding about how progress in learning number occurs...Immediate benefit would be seen if teachers' confidence in their own mathematical competence could be improved.' (pp21-22)*

The recent annual report of HMCI (Ofsted, 1995a) repeats these points:

> *'The foundation laid in KS1, however, was not always consolidated. Too few KS2 pupils were able, for example, to recall their tables, compute with sufficient speed and accuracy, apply their knowledge in investigative work.' (pp16-17).*

Preliminary results from KS2 national tests (SCAA, 1995) suggest that whereas the distribution of pupil achievement at KS1 has been roughly in line with expectations, the distribution at KS2 is below expectation with 35% below level 4 (the expected average level at the end of KS2) and only 18% above. This finding is consistent with one of the main findings from the review of mathematics inspection findings 1993/94:

> *'The primary schools give a lot of attention to routine number work and standards in Key Stage 1 are often good and sound overall. However, in Key Stage 2 standards are less satisfactory mainly because the rate of progress is too slow and misconceptions and errors are not addressed. Too many pupils are unable to recall important number facts or to compute with sufficient speed and accuracy.' (Ofsted, 1995b) p3)*

1.2.2 Research into pupils knowledge of and facility with numbers, number relations and operations and their ability to apply this knowledge

While young children display some competencies with counting strategies before coming to school, it seems that their understandings of the purposes behind such skills do not always accord with how adults perceive the task (Munn, 1994). A key role of the early years teacher is to help young pupils move to understand counting as a purposeful activity through activities that require the application of counting skills (Aubrey, 1993).

Studies of arithmetical methods used by 5-7 year-olds (e.g. Carpenter & Moser, 1984) and 7- to 12- year-olds (e.g. Steffe, 1983; Gray, 1991) demonstrate that numerate pupils have a range of alternative strategies to draw on, based on both: 'knowing by heart'—recall of some number facts (for example 5 + 5 = 10) and 'figuring out'—deriving or deducing other number facts on the basis of the known facts (for example 5 + 6 must be one more than 5 + 5).

It seems that pupils with access to *both* recalled and deduced number facts make more progress because each approach supports the other:

- deducing number facts helps pupils commit more facts to memory, and
- recalled facts help expand the range of strategies for deriving facts.

On the other hand, lower-attaining pupils rely mainly on counting strategies based on objects (fingers or counters) or representations of objects (Denvir, 1984). Other strong evidence in research indicates that, across all years of schooling, some pupils do not progress far beyond developing arithmetic techniques that rely on simple addition skills, such as 'counting on' or relying on repeated addition for multiplication (Hart, 1981). For some lower-attaining pupils it appears that over-dependence on counting methods, while leading (eventually!) to a correct result, removes the need to commit number facts to memory, which in turn limits their development in becoming numerate (Anghileri & Johnson, 1992).

Research is beginning to show that the use of practical materials is not necessarily the best way of encouraging pupils to overcome such difficulties and be able to abstract mathematical concepts and develop mental strategies. For example it has been shown that from an early age children can operate with small numbers when they are linked to objects (e.g. two elephants and two more elephants), but even after immediately being 'tuned in' to the real-world/mathematics link, they find it difficult spontaneously to put into a context numbers presented in an abstract form (Hughes, 1986). Furthermore, there is little evidence to suggest that facility in particularising an abstract context to support progress in tackling a problem gets any better with age.

The Children's Mathematical Frameworks project (Hart, Johnson, Brown, Dickson, & Clarkson, 1989) confirmed this feature and showed more generally that the link between practical work and the move to formal symbolic mathematics is often tenuous. While teachers used practical work as a justification for formal methods, pupils often failed to make any firm or lasting connections between the practical and abstract. A detailed study and analysis of a group of pupils learning place value from tens and units blocks (Walkerdine, 1988) indicated that the blocks themselves served only as a vehicle for teacher talk—the learning came about from the way the teacher talked about and handled the blocks, rather than through the pupils' own discoveries .

Research shows that increased confidence and competence in knowledge is closely bound up with the ability to apply that knowledge: the relationship between 'knowing' and 'applying' is cyclical rather than linear (Nickerson, 1988). Comparisons between experts and novices have shown clear differences in the way that they each tackle problems (Campione, Brown, & Connell, 1989). For example, a study of primary school pupils (aged from 4 to 9) looked at problem solving strategies across the age range. Typically the younger pupils did not attack the problem strategically while in contrast, the older pupils spent a considerable time planning their approach. It seems that the older pupils had a far clearer perception of the benefits of 'thinking about their thinking' (Karmiloff-Smith, 1979).

The ability to perform written calculations accurately relies on understanding of the principles underlying algorithms (Ginsburg, 1977). Brown & Van Lehn (1982) demonstrated that long term retention of written algorithms depended on the deployment of reasoning depending on understanding of the conceptual basis of written methods in order to retrieve steps in the process that had been forgotten.

How this will inform the starting point for this project

A main message from research is that while practical work and 'real' contexts can be useful, they need to be chosen carefully, and accompanied by careful dialogue with pupils to establish the extent of their understanding and to help pupils develop links between the practical and the abstract. The organisational skills and pedagogic knowledge which effective teachers have that enable them to engage in such dialogue will be one focus of attention in the project.

In both identifying effective teachers and observing their practice attention will be paid to the use of activities and intervention strategies that focus on helping pupils both commit some number facts to memory and develop strategic approaches to deducing other number facts.

In the context of National Curriculum mathematics, this would suggest that learning activities which focus on narrow skills are less likely to be successful than those that require pupils to integrate ideas, especially between understanding and skills in number (Ma2) and the 'using and applying' strategies of problem-solving, communicating and reasoning(Ma1). The extent to which this is the case in the classes of effective teachers will be examined within this project.

1.2.3 Research into effective teachers

Shulman (1987) suggests that there are several types of understanding and knowledge that impact on practice:

a content knowledge/general pedagogical knowledge/ curriculum knowledge/pedagogical content knowledge

b knowledge of learners and their characteristics

c knowledge of educational contexts

d knowledge of educational ends, purposes and values and their philosophical and historical grounds.

While research has begun to examine the actual impact of the above forms of knowledge and understanding on classroom practice, many of the findings present a deficit model of teacher knowledge. For example, in terms of mathematical content knowledge, research shows that many teachers' own mathematical understandings are limited (Kennedy, 1991). Observation of lessons given by such teachers suggests that this lack of mathematical knowledge leads to a concentration on inculcation of disconnected algorithmic skills. On the basis of such findings it has been argued that improving teachers' own mathematical knowledge base will lead to better teaching (Alexander, Rose, & Woodhead, 1992).

While this may be a logical conclusion of such research, there appears to be little research to support this conclusion in practice: research may demonstrate that teachers with limited mathematical knowledge are less effective, but there is scant evidence that teachers with sound mathematical knowledge are actually more effective. Where evidence for the importance of mathematical subject knowledge is presented it tends to be based on the effect on classroom practice rather than pupil outcomes. For example a study of post-graduates in training demonstrated a link between subject knowledge and style of teaching but no measure was made of pupil outcomes (Bennett & Turner-Bisset, 1993). However Leinhardt, Putnam, Stein, & Baxter (1991) in their analysis of good and poor mathematics teaching concluded that subject knowledge impacted in several ways. Teacher's mental plans for lessons were dependent upon their familiarity with the content to be taught (c.f. Borko, Livingston, McCaleb, & Mauro, 1988) and the questions asked and explanations offered to pupils reflected the teachers subject knowledge.

Research on teachers' knowledge of learners also shows that this knowledge is often not well grounded. One study suggests that it is easy to ‘foreclose’ on pupils—to jump to conclusions about a pupil’s difficulty, either on the basis of limited information or by drawing on past experience (Bennett, Desforges, Cockburn, & Wilkinson, 1984).

However it is also clear that teachers can improve their diagnostic and remediation skills. Crooks (1988) showed that teachers trained in diagnosis knew more about the processes that individual pupils used to solve problems, and their pupils did better in number knowledge, understanding, problem solving, and confidence. A control group tended only to explain problem-solving processes to pupils or just observe their pupils' solutions. Knowledgeable teachers have been demonstrated to spend more time questioning their pupils about mathematical processes and listening to their responses (Peterson, Carpenter, & Fennema, 1989). This suggests that it may be more important to have a sound grasp of pedagogical content knowledge than subject content knowledge (Carpenter, Fennema, Peterson, & Carey, 1988).

Other studies point to the importance of establishing of a particular classroom culture (Cobb, 1986), raising the issue of teachers' belief systems about mathematical knowledge and how it is generated and learnt. It may be that beliefs about the nature of the subject are more influential than mathematical subject knowledge per se (Lerman, 1990; Thompson, 1984).

Many studies, particularly in the USA, focus on effective classroom practice and routines (Berliner, 1986) but research demonstrates the difficulty that teacher experience in adopting new practices without an appreciation of and belief in the underlying principles (Alexander, 1992). Further, teachers may have adopted the rhetoric of 'good' practice in teaching mathematics without changes to their actual practices (Desforges & Cockburn, 1987).

How this will inform the starting point for this project

Research findings suggest that effective teachers need to have good 'mental maps' of pupil lines of development. The proposed research will examine the extent and nature of such understandings. The project will also examine the role that diagnostic assessment plays in the classes of effective teachers and the extent to which they make use of it in developing pupils' understanding.

By identifying effective teachers primarily on the basis of pupil levels of attainment, claims to outcomes being dependent upon particular types of knowledge can be examined. Further the effect of interplay between different forms of knowledge as set out in Shulman's taxonomy can be explored.

The research on the links between knowledge, beliefs and practice suggests that a mix of techniques to elicit teachers' knowledge and understanding backed up by classroom observation to examine actual practices in required. This means working with a relatively small sample of teachers within limited resources but we believe that detailed qualitative data supported by appropriate quantitative data can provide the sort of insight that this work demands. This will build upon methods and analyses previously developed by the project director in examining principles and practices in the teaching of Ma1 (Askew, Brown, Johnson, Millet, Prestage, & Walsh, 1993).

1.2.4 Research into means of change and professional development

Many writers have drawn attention to the difficulties of implementing lasting changes in an educational system. Fullan (1991), in a comprehensive review of empirical studies of educational change, identifies four main factors:

1. active initiation and participation
2. pressure and support
3. changes in behaviour and beliefs
4. the overriding problem of ownership.

In relation to the second factor of **pressure and support**, he summarises the results of several studies as 'the degree of change was strongly related to the extent to which teachers *interact* with each other and others providing technical help', and notes that 'collegiality ... was a strong indicator of implementation success' (p. 131).

Certainly much literature in this country has supported the need for school-centred or school-based professional development(e.g. Easen, 1985; Bell, 1991; Bolam, 1982). Hopkins (1989) notes the power of linking professional development to school improvement rather than the previous 'ubiquitous "one-shot" inservice workshops that have proven to be so demonstrably inefficient'(p.86). Mortimore, Sammons, Stoll, Lewis, & Ecob (1988), in a study of a large sample of ILEA junior schools, demonstrate improved attainment in mathematics and other subjects in schools with effective leadership and staff collaboration. Nias (1985) also points to the need for a 'reference group', and others stress the need for this to support teachers due to the uncertainty and risk involved in professional change (e.g. Erault, 1982; Biggs, 1983; Critchley & Casey, 1984; Pinner & Shuard, 1985; Pirie, 1987). Nolder (1992) demonstrates that while the reference group can be outside the school, in such cases the degree of change attained by a teacher is gradually eroded as within-school change is negotiated.

This collegial aspect is in contrast to much traditional INSET activity which Halpin, Croll, & Redman (1990) found to be perceived by teachers as contributing to their personal knowledge and classroom skills and to increased attainment by their pupils, rather than to any school goals.

However Smyth (1989) points out that in-school development is less likely 'to founder on the rocks of transference (to a different context), ownership(by a particular group) or adoption (by unwilling participants)' (p 219). The question of

ownership also links with Fullan's fourth factor and his findings that 'ownership in the sense of clarity, skill and commitment is a progressive process' (p92). Many authors in addressing the importance of ownership emphasise Fullan's first finding about the need for **active participation**, stressing the effectiveness of bringing about professional development by working in classrooms with teachers who are involved in setting their own agendas (e.g. Joyce & Showers, 1980; Biggs, 1983; Straker, 1988; Elliott, 1989; Day, 1989; Cobb, Yackel, & Wood, 1988; Jaworski, 1991; Nolder, 1992; Monteiro, 1994).

Much of the work referred to, deriving often from the notion of 'reflective practitioner' coined by Schon (1983) has stressed the importance of reflective activity, either collaboratively through collegial interaction or in the context of external agencies working with teachers in classrooms. The aim in both cases is to assist teachers to first make explicit their tacit beliefs in order that they may be examined and re-assessed. Erault (1982) notes the importance of this given that the tacit nature of knowledge-in-action means that there are often inconsistencies, of which teachers are unaware, between their 'espoused' views about good practice, expressed in interviews and essays, and their classroom practice. Desforges & Cockburn (1987) give examples of this dissonance but, questionably, relate it to the constraints of classrooms.

Erault also notes the difficulties teachers experience in evaluating INSET activities since they 'may not be able to state what they have learned or even, in some cases, whether they have learned'(p.6). Bolam (1982) points out for the same reason the difficulties of using questionnaires to evaluate courses.

Ernest (1989) also emphasises the importance of reflective activity in changing teachers' beliefs, thus linking with Fullan's third factor concerning changes in **beliefs** to changes in practice. Ernest follows researchers such as Davis (1967), Thompson (1984), Cooney (1985) and Lerman (1989) in stressing the relevance of teachers' beliefs: 'Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change '(p.249). However Fullan notes that research suggests that the process of changing beliefs and practices is not linear, rather 'In many cases, changes in behavior precede rather than follow changes in belief'(p.91), thus echoing the views of Cobb et al. (1988) that' beliefs and practices are dialectically related'(p.24) . Cooney (1988) also notes the changes in the mode of use of textbooks by teachers from instrumental, to subjective, to fundamental, relates to the extent to which they link these to their own beliefs, with the most sophisticated use being achieved only where teachers have a coherent underlying philosophy of mathematics education.

Cooney's work links with that of other researchers like Pinner and Shuard (*op. cit.*.) House & Lapan (1989), Elliott (*op. cit.*) and Weller (1992), who have drawn attention to developments which takes place over the 'life-cycle' of an individual teacher, in moving e.g. from being a 'restricted professional' to an extended professional' (Hoyle, 1974).

How this will inform the starting point for this project

The findings quoted in this section suggest that effective strategies for professional development need to incorporate the following factors:

- emphasising reflective activity through interaction and discussion
- encouraging ownership and active participation
- linking with school improvement policies
- establishing teacher support groups
- focusing on classroom practice, possibly with external support
- relating changes in practice with changes in beliefs.

In relation to methods to be used in the research, the evidence of discrepancies between practice and espoused views highlighted by Erault (1982) suggests that observation needs to be used alongside interviews to determine factors underlying effectiveness. The work of Erault (and Bolam, 1982) in drawing attention to the difficulties of asking teachers to evaluate the contribution of training to their professional development suggests that different approaches to this need to be made, with triangulation to establish validity.

The work of Mortimore et al. (1988) suggesting that increased attainment in mathematics in ILEA is related to whole-school factors will be used to justify the selection of effective teachers through prior selection of effective schools.

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